



ADVANCED MIMO CHANNEL ESTIMATION TECHNIQUES: A COMPREHENSIVE REVIEW OF METHODS, ALGORITHMS, AND PERFORMANCE OPTIMIZATION

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Abstract

The promising characteristic features of Multiple-Input Multiple-Output (MIMO) systems rely on the knowledge of the channel state information (CSI) for coherent signal data detection. The determination of channel state information is achieved using various conventional estimation techniques such as pilot-aided, blind, and semi-blind channel estimation techniques. Obtaining accurate channel state information in MIMO systems is significant tasks upon which system performance depends. This paper presents a comprehensive review of various MIMO channel estimation techniques presented in literature from conventional techniques to more recent deep neural network-based techniques. Various ways of pilot arrangement and complexity reduction techniques are discussed. Furthermore, the key performance indicators in MIMO channel estimation, various algorithms applied in channel estimation and the impact of outdated CSI with its causes are also presented. The recent improvements on the conventional techniques with its impact on the key performance indicators in communication systems such as 5G and beyond Networks, Millimeter-wave Communications, and Massive MIMO system were also reviewed. Accurately estimating wireless channel condition makes signal transmission adaptive leading to optimal performance in transmission and decoding of signals.

1.0 INTRODUCTION

As wireless communication systems continue to evolve with increased demand for data rates, MIMO technology has emerged as a key enabler for achieving enhanced spectral efficiency and improved system performance. MIMO systems support multiple transmitting and receiving antennas for wireless communication. In a MIMO channel which defines the wireless link between the transmitter (Tx) and the receiver (Rx) pairs as presented in Figure 1, multiple spatial streams are transmitted simultaneously over the same frequency band. It leverages the increased spatial freedom to increase capacity, improve reliability, extend coverage and mitigate interference. MIMO channel is characterized by a channel matrix that captures the multipath propagation effects, such as reflections, scattering, and diffraction, which cause the transmitted signals to arrive at the receiver antennas with different amplitudes, phases, and delays. The MIMO system leverages this multipath propagation to improve performance. This matrix represents the complex-valued fading coefficients between each transmit-receive antenna pair. In a

MIMO channel, the mathematical expression for the input-output relationship can be expressed as in Equation 1 [1].

$$y(t) = H(t) * s(t) + n(t) \tag{1}$$

Where, $s(t)$ is the transmitted signal, $y(t)$ is the received signal, $n(t)$ is additive white Gaussian noise (AWGN), $H(t)$ is an N_{RX} by M_{TX} channel impulse response matrix and $(*)$ denotes convolution

Where:

$y(t)$ is the received signal vector:

$$y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} \tag{2}$$

$H(t)$ is the 2×2 channel matrix:

$$H(t) = \begin{bmatrix} h_{11}(t) & h_{12}(t) \\ h_{21}(t) & h_{22}(t) \end{bmatrix} \tag{3}$$

$s(t)$ is the transmitted signal vector:

$$s(t) = \begin{bmatrix} s_1(t) \\ s_2(t) \end{bmatrix} \tag{4}$$

$n(t)$ is the noise vector:

$$n(t) = \begin{bmatrix} n_1(t) \\ n_2(t) \end{bmatrix} \tag{5}$$

the input-output relationship of a 2×2 MIMO system becomes [2]:

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} h_{11}(t) & h_{12}(t) \\ h_{21}(t) & h_{22}(t) \end{bmatrix} * \begin{bmatrix} s_1(t) \\ s_2(t) \end{bmatrix} + \begin{bmatrix} n_1(t) \\ n_2(t) \end{bmatrix} \tag{6}$$

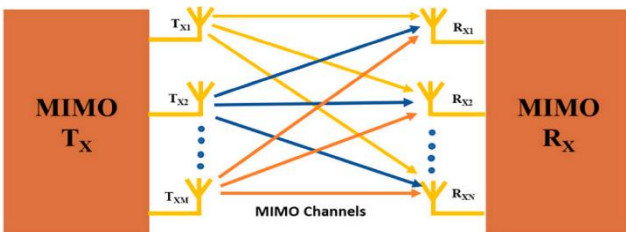


Figure 1: MIMO Transmitter, Channel, Receiver [3]

In a MIMO system equipped with T transmit and R receive antennas system as presented in Figure 2, $T \times R$ sub-channels are obtainable between the transmitter and receiver pair. In general, each sub-channel exhibits a selective fading leading to a complex interaction with the transmitted signal due to diffractions, reflections and signal scattering [1]. The source signal is encoded using vector encoder, modulated and transmitted using multiple antennas. Different techniques used for transmission and reception of signals in MIMO systems depends on the availability of CSI at the transmitter as well as the reliability, complexity and bit rate of the system. Transmission with the assumption that channel condition is known is optimally done using the low complexity Space-time coding (STC) or spatial multiplexing which offers high diversity gain and average bit rate [4][5]. However, when the transmitter

has access to the CSI, it utilizes beamforming techniques for transmitting a data stream across the wireless link exploiting the array gain to significantly mitigate the effect of channel fading [6]. This approach enhances both the system's spectral efficiency and its resilience, thereby improving its overall performance.

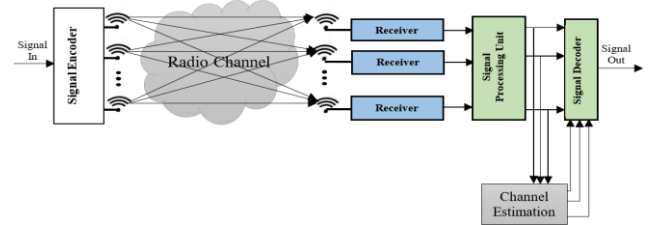


Figure 2: MIMO system model

2.0 CHANNEL ESTIMATION (CE) IN MIMO SYSTEMS

The inherent advantages of MIMO in wireless communication systems drew huge attention from the research community. However, these benefits can be fully achieved only when the channel state information (CSI) is known and available at the transmitter. This is because MIMO systems rely on the knowledge of channel state information for coherent detection and effective decoding of the received signal [7]. The significance of channel estimation is dependent on the complexity of the interactions between transmitted signals and the environment in a MIMO wireless system. It helps to compensate for the effects of channels on the transmitted data to achieve the benefits stated above. The CSI can be known through channel estimation. The efficiency of signal detection is rated based on the accuracy of the acquired CSI [7]. Channel Estimation is the technique employed for determining channel parameters from the received signal. On the other hand, it is the practice of characterizing or evaluating the influence of the physical channel on the input data signals. The parameters are used to make transmission responsive to current channel conditions making adaptive and reliable communication achievable with high data rates.

2.1 Key Performance Indicators (KPIs) in MIMO CE Techniques

Evaluating MIMO channel estimation techniques based on significant KPIs enables researchers to identify strengths, weaknesses, and trade-offs between different channel estimation techniques, leading to enhanced performance in MIMO systems. In Table 2, the performance evaluation of some recent research work on channel estimation based on some of the key



performance indicators (KPIs) given in Figure 3 are summarized.

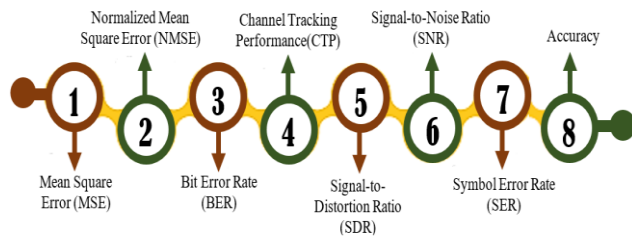


Figure 3: Key performance indicators (KPIs) in MIMO CE techniques

- i. **Mean Square Error:** MSE is determined by comparing the accuracy of the estimated channel to the true channel. The lower the MSE value, the higher the accuracy of the estimation. Among other algorithms, Minimum Mean Square Error (MMSE) algorithm frequently obtains the lowest MSE value [8][9]. In the MMSE algorithm, noise is accounted for which enhances the accuracy of the estimates obtained even in cases with low SNR.
- ii. **Normalized Mean Square Error:** NMSE is a metric that quantifies the deviation between the true channel and the projected state of the channel. NMSE determines the extent of the channel variations by normalizing the value of the MSE with the exponents of the coefficients of the true channel. The lower the NMSE, the better the performance of the channel estimation technique [10].
- iii. **Bit Error Rate:** BER indicates the accuracy of data received per time. Optimal performances are characterized by low BER. The Linear Minimum Mean Square Error (LMMSE) estimation algorithm is efficient in minimizing BER but it requires the knowledge of both noise and channel statistics [11].
- iv. **Channel Tracking Performance:** CTP determines the adaptability of channel estimation techniques to dynamic wireless channels [12]. Good channel tracking improves the quality of channel estimation and ensures continuous and reliable communication in varying channel conditions [13].
- v. **Signal-to-Distortion Ratio:** SDR is a crucial performance indicator that shows the cumulative error existing amid reference and estimated channel [14]. An optimal performance in channel estimations is characterized by a higher SDR.
- vi. **Signal-to-Noise Ratio:** the performance of channel estimation techniques in wireless systems are critically evaluated based on SNR. It quantifies the ratio of signal to noise power of received signal in a given channel [15]. At higher SNR, better and

accurate channel estimates are achievable leading to improved overall performance.

- vii. **Symbol Error Rate:** with respect to channel imperfections and noise variable, SER indicates the measure of accuracy of the received symbols with respect to the transmitted symbols. A lower SER indicates an optimal channel estimation performance. BER and SER is the key performance indicator for actual signals in AWGN channel [16].
- viii. **Accuracy:** Accuracy indicates the measure of precision of the estimated channel parameters to the true channel parameters. Accurate estimation achieves lower symbol error rate (SER) or bit error rate (BER).

2.2 Classification of MIMO Channel Estimation Techniques

There are various ways for estimating MIMO wireless channel as presented in Figure 4. MIMO channel estimation techniques can be classified based on the use of pilot signal [17], nature of the channel [18] or application of machine learning [19]. Consequently, MIMO channel estimation techniques could be classified as Pilot-based channel estimation technique in which a known pilot signals are multiplexed with the data signal to be used in estimating the channel condition at the receiver, or Blind channel estimation technique where only the mathematical and the statistical properties of the transmitted signal is extracted for channel condition estimation [20]. However, some special cases exist where few pilot signals are used together with the mathematical and the statistical properties of the transmitted signal to estimate the channel conditions. It is known as semi-blind channel estimation technique [20]. Moreover, MIMO channel estimation technique have also been categorized in the literature as compressed sensing channel estimation techniques which rely on the sparsity of the channel to extract the CSI and deep-learning channel estimation technique which leverages artificial neural network models to predict more accurate channel statistics. To acquire a more reliable and accurate estimates, the deep-learning technique goes through pre-training phase, learning phase and testing phase.

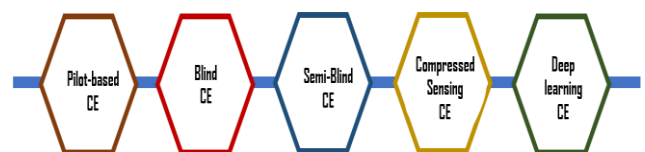


Figure 4: Classification of MIMO channel estimation techniques



2.2.1 Pilot-based channel estimation (PBCE) technique

Pilot-aided channel estimation techniques fundamentally depend on using a known evenly distributed pilot sequence multiplexed with data symbols to estimate the channel condition at the receiver [20]. Having the CSI at the transmitter improves the performance of the system. The estimation could be achieved under time and frequency domains, depending on the pilot arrangement employed [21]. Pilot-aided channel estimation techniques are less computationally intensive compared to blind techniques. However, the pilot signal consumes the scarcely available bandwidth which significantly degrades the spectral efficiency and the useful data rate with increased overhead. The accuracy of PBCE can be improved with higher density of pilot signals but with significant performance loss [20]. The performance of Pilot-based CE algorithms is significantly limited in massive MIMO systems due to the linear relationship between the pilot overhead and the size of the MIMO system.

In pilot-based MIMO channel estimation, pilot symbols are multiplexed with the data symbols for estimating the wireless channel characteristics. The way the pilot signals are arranged plays a significant role in improving the accuracy of the MIMO channel estimation. Pilot signals can be arranged in two fundamental ways for channel estimation: block-type and comb-type pilot arrangements as presented in Figures 5a and 5b.

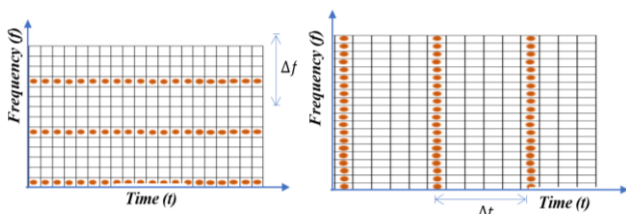


Figure 5a: Comb-type Pilot structure

Figure 5b: Block-type Pilot structure

In the block type, the pilots are sent periodically in the time-domain in all the sub-carriers. Its computation requires interpolation only between the pilot blocks since all the sub-carriers are used for pilot transmission. Predominantly, it is suitable for slow-fading channels and relatively insensitive to frequency selectivity [21]. The comb-type pilot arrangement uses all the sub-carriers for pilot transmission within a specific period of frequency. The pilot arrangements are evenly distributed within each subcarrier. The channel estimations are carried out in the pilot subcarrier channels and then interpolation techniques are applied for estimation at the data subcarrier

channels. Unlike the block-type, the comb-type pilot signal structure responds to changes in frequency and as well robust to fast-fading channels [17].

There exist two major challenges in designing efficient pilot-based channel state estimators for MIMO OFDM systems: Arrangement of pilot signals and the computational complexity incurred from complex matrices inversion [22]. The solution to these estimator design challenges will offer both low complexity and good channel tracking performance. Complexity can be reduced by Interpolation [23] or by reducing the number of procedures involved in the matrix inversion by applying Neumann Series (NS) expansion [22] or iterative techniques [24]. Pilot-based techniques are inherently limited in spectrum utilization. However, [25] proposed a new pilot sequence based on comb-type pilot arrangement to improve the spectral efficiency of LS algorithm. Furthermore, a pilot-based scheme aimed at reducing impact of pilot contamination is presented in [26]. The scheme used Chu sequence pilot design with MMSE algorithm in a block-fading channel model. The channel estimation error is significantly reduced leading to low NMSE which improved the system overall performance. The accuracy of channel estimation can be significantly influenced by the selection of the estimation threshold; hence Single Pilot Aided (SPA) and Multiple Pilots Aided (MPA) channel estimation for Affine frequency division multiplexing (AFDM) is proposed in [7]. At optimal estimation threshold, the accuracy of the SPA estimation increases, indicating lower BER as SNR increases [7]. Using LMMSE algorithm, the BER performance of Affine frequency division multiplexing (AFDM) with estimated CSI is comparable to AFDM with perfect CSI [7].

2.2.2 Blind channel estimation technique

The blind estimation technique exploits the mathematical or statistical properties of transmitted data exclusively to estimate the channel parameters [17]. This technique drew much attention from the research community on the account of its potential to improve spectral efficiency without acquiring extra pilot overhead [27]. However, this technique is impracticable as a result of its high computational complexity. Moreover, Blind Channel Estimation is only applicable to slow time varying channels due to its need for a huge number of received symbols that is required to extract the statistical variables of the received signal for channel estimation.



A less complex blind channel estimation technique with full rated encoder is presented in [28] to achieve a fast convergence and improved spectral efficiency using square-root operations. It could reduce the noise power to half; however, the square-root operation introduces phase ambiguity which could be eliminated by sending one more pilot sequence instead of applying interleaved symbol mapping. Correntropy Independent Component Analysis (ICA) [29] and Second-Order Blind Identification (SOBI) [30] were applied for blind channel estimation in the statistical domain. The correntropy ICA algorithm is an improvement on SOBI which depends only on second-order statistics [31]. MSE and SDR performance of ICA is superior to that of SOBI even at higher SNR, this is because, beyond second-order statistical variables known to SOBI, ICA can extract higher-order statistical information of the source signal [31].

2.2.3 Semi-blind channel estimation technique

Semi-blind channel estimation integrates some features of both pilot-based and blind channel estimation techniques to improve channel estimation accuracy in MIMO communication systems. The approach uses few pilot signals to produce initial channel state information, while additional statistical parameters extracted from the transmitted information signal are exploited to improve the accuracy of the channel estimates [32]. The robustness of pilot-based channel estimation techniques and the spectral efficiency of blind estimation technique due to low signal overhead is harnessed by this hybrid technique, making it more applicable in scenarios where either pilot-based technique or blind technique is inefficient. The superiority of the semi-blind channel estimation scheme over the blind estimation scheme is validated in [33] using perturbation analysis. The author proposed an approach that integrates a blind algorithm based on linear prediction into the LS scheme. The result of the proposed scheme has a significant improvement in MSE and BER compared to the LS method.

2.2.4 Compressive sensing (CS) channel estimation technique

Compressed Sensing provides an attractive alternative to conventional channel estimation techniques in modern communication systems. The sparsity of MIMO channels due to its multipath characteristics have not been exploited by conventional CE techniques [18]. However, Compressive sensing channel estimation technique has the capacity to leverage the sparsity of MIMO channels to estimate the wireless channel response with few measurements.

It offers the potential for more efficient and accurate channel estimation, particularly in scenarios with sparsity or compressibility in the channel response such as in frequency-selective fading channels because the sparsity is uniform within the coherent period. This technique has the advantage of improved spectral efficiency with reduced pilot overhead. However, it introduces additional computational complexity due to the need for sparse signal recovery algorithms which optimally adjust the trade-off between channel tracking and noise mitigation. The effectiveness of compressed sensing algorithms heavily relies on the sparsity level of the channels being estimated. Hence, Speed and accuracy is a critical requirement in CS CE to avoid generating an invalid CSI [18].

An outstanding performance is achieved in [34] by leveraging delay-Doppler sparsity to reduce the required number of pilot signal for CE even at a rate much less than Shannon rate. Using classical basis pursuit algorithm, a superior performance in MSE with improved spectral efficiency than LS is obtained in high SNR regime. However, exploiting the characteristic structured sparsity in MIMO channels using sparsity update CoSaMP proposed in [35] an optimal MSE performance is achieved in both low and high SNR regime. The scheme has the advantage of adaptively acquiring the channel's sparsity level unlike CoSaMP.

2.2.5 Deep learning-based channel estimation technique

Due to the non-uniform distribution of CSI, Neural Networks (NN) are more efficient in dealing with channel imperfections at a faster rate without extra power constraints compared to conventional techniques in wireless communications. The deficiencies of multi-user wireless communications such as noise, fading and high BER can be mitigated by the application of artificial Neural Networks (ANN) in channel estimation [36]. In addition, soft-computing applications such as Deep Neural Networks (DNN) in MIMO wireless communications, promises a robust and effective means for more accurate channel estimation. DNN comprises several layers of non-linear processing units. The input units known as neurons are interconnected in successive layers. The input data propagates from the input layer, across the hidden intermediate layers and finally received from the output layer. The output in each hidden layer is used as input in the consecutive layer.

Deep convolution neural network (CNN) is deployed in [37] to effectively improve the accuracy of channel



estimation technique with reduced pilot overhead at the cost of increased complexity. Similarly, NN and CNN based techniques proposed in [38] improved the training speed and accuracy of channel estimation respectively. CNN-based scheme with larger filter size, effectively removed the effect of pilot contamination and outperformed the faster NN-based estimator in terms of loss value. However, both schemes have a better NMSE performance than LS and covariance estimation techniques. Furthermore, modeling channel estimation as a denoising problem using deep residual learning (DReL) with a CNN as in [39] and [40] helps to obtain accurate channel estimates with optimal performance in NMSE as MMSE. While one dimensional denoising block is used in [39] to obtain a near optimal performance, three dimensional (3D) denoising block used in [40] exploits both temporal and spatial correlations of the pilot signal at the receiver to achieve more accurate estimates.

2.2.6 Kalman filter and its variants

The Kalman filter is a mathematical algorithm that recursively estimates the state of a linear dynamic system based on a sequence of noisy measurements. The filter maintains two main estimates: firstly, the predicted state, which is acquired from the initial state estimate and system dynamics; and secondly, the corrected or updated state, which is obtained by integrating the data from the new measurement. The state estimates from each stage are obtained by applying a system of control through feedback [41].

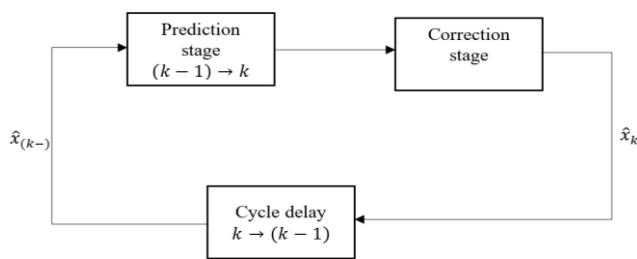


Figure 6: Kalman framework cycle

In the Kalman filter framework presented in Figure 6, the tracking equations are divided into two primary components: prediction equations and correction equations which are applied in the prediction stage and the correction stage respectively. The prediction equations utilize carefully selected models to project the current estimate and error covariance estimate in time, thereby executing the prediction stage to obtain for the next step the priori estimates. On the other hand, the correction equations are responsible for addressing errors and inaccuracies that emerge during the estimation process. This is achieved by integrating the acquired measurements into the priori estimates to obtain a new improved posteriori estimate.

Due to various limitations encountered in the application of Kalman filter in different domains, several variants specialized to adapt and tackle the limitations and offer performance improvements are proposed in the literature. These intended adaptations help to address definite challenges such as non-linear system dynamics, computational efficiency, adaptability and non-normal noise distributions. Variants like the Extended Kalman Filter (EKF) [42] and Unscented Kalman Filter (UKF) [43] extend the standard linear Gaussian framework to address nonlinear systems through different approaches. Others such as Dual Kalman Filter (DKF) tend to achieve a balance between adaptability and complexity of computation [44] whereas Robust Kalman Filter (RKF) accounts extensively for parameter uncertainties [8]. Each established variant offers its peculiar set of improvements and trade-offs. In addition, the challenge of latency and memory deficiency in forward-backward Kalman filter (FBKF) is overcome with forward-only Kalman (FOKF) filter [45]. One major advantage of the variants is that it makes Kalman filter family versatile and applicable to widespread practical applications. The comparison of the variants of Kalman filter is presented in Table 1.

Table 1: Comparison of the variants of Kalman

Citation	Variants	Target	System	Trade off	Improvement
[46,42,47]	EKF	Linearization	Nonlinear	Complexity	Performance
[43,48]	UKF	No linearization	Nonlinear	Complexity	Accuracy
[44,49]	DKF	Adaptability/complexity	Nonlinear	Complexity	Adaptability
[50]	RKF	Parameter Uncertainties	Linear	Performance	Robustness
[51]	FBKF	Higher accuracy	Linear	complexity	Accuracy
[45]	FOKF	Latency/complexity	Linear	Accuracy	Speed

2.3 Complexity Reduction Techniques in Channel Estimation

Matrix inversion is the major factor that scales up the computational intensity of channel estimation techniques, necessitating the need for complexity

reduction techniques. Researchers have developed several techniques including interpolation, rank reduction, matrix convergence, and orthogonal matrix triangulation as given in Figure 7. The interpolation technique minimizes the number of matrix inversion



by extracting channel information at pilot subcarriers with comb pilot structure. Interpolation technique offers optimal performance at lower Doppler frequencies [52]. Complexity reduction is also achieved through Rank reduction by decomposing the channel autocorrelation matrix using singular value decomposition (SVD) [53] or eigenvalue decomposition (EVD) [54]. SVD and EVD techniques are used to avoid complex inverse matrix operations leading to improved performance of channel estimation techniques. In addition, matrices convergence technique reduces the number of operations involved in the matrix inversion by applying approximation technique such as Neumann Series (NS) expansion [55]. Matrix factorization technique such as Orthogonal Matrix Triangularization scales down a full-rank matrix to a less complex form [56]. It is superior to other techniques such as SVD because it achieves numerical stability by reducing errors introduced by machine round-offs [57].

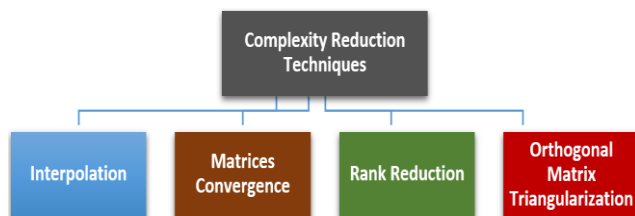


Figure 7: Computational complexity reduction techniques

3.0 THE CHALLENGE OF OUTDATED CSI

In wireless communication systems, accurate knowledge of the channel state is vital for efficient signal transmission and reception. However, due to the dynamic nature of the wireless channel, the CSI quickly becomes outdated. This can lead to performance degradation, as the system makes decisions based on inaccurate information. This challenge is caused by many factors such as feedback delay, network user mobility, environmental dynamics, short channel coherent time, and operating frequency of the system. Feedback delay is the time taken to transmit CSI from the receiver to the transmitter [58]. Any variation of the channel within this time makes the CSI to be outdated. In this scenario, the transmitter makes the next transmission decision based on the received CSI which is already outdated, leading to degradation in the performance of the system. To keep more accurate CSI, the need to reduce the feedback delay is necessary. This keeps the CSI current and reduces the errors due to aging [59]. In addition, user mobility causes the wireless channel condition to change more rapidly. This variation in

channel characteristics arises due to the change in the distance between the transmitter and receiver leading to channel fading, path loss and Doppler shift.

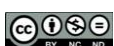
The channel condition is acquired at a given moment in time, capturing the characteristic condition of the channel; any subsequent change due to the movement of network users renders the already acquired CSI outdated. This is a critical challenge especially in scenarios with high-speed mobility. In vehicular networks for instance, the channel condition can vary within milliseconds, potentially rendering the CSI outdated shortly after it is obtained. Environmental changes such as weather and terrain also affect the rate at which CSI gets outdated. When these changes occur frequently and rapidly, the channel characteristics is updated concurrently. Consequently, an accurate CSI in a given environmental condition becomes obsolete in subsequent change in environmental condition. Therefore, any transmission made based on the acquired CSI produces suboptimal performance when the environmental variation occurs. Short channel coherent time, also lead to higher expiration rates of CSI [60]. In high Doppler spread channels with short channel coherence time; the CSI must be updated more regularly to maintain accuracy [60]. Finally, systems operating at higher frequencies are more susceptible to Doppler shifts and multi-path fading leading to quick expiration of CSI. It is necessary for system designers develop robust strategies to mitigate the effect of CSI aging to improve reliability and overall performance of communication systems.



Figure 8: Applicable algorithms in MIMO channel estimation technique

4.0 APPLICABLE ALGORITHMS IN MIMO CHANNEL ESTIMATION TECHNIQUE

There are several algorithms used in MIMO channel estimation as given in Figure 8. Each is designed to address different challenges in estimating the MIMO channel. The choice of the best algorithm depends on various factors, including the specific MIMO channel characteristics, system requirements, and implementation constraints. Proper evaluation and simulation are necessary to determine the optimal channel estimation technique for a given scenario. Presented



in Figure 8 are some frequently used algorithms and the techniques they perform well in:

- i. **Least Squares:** LS algorithm is mathematical algorithm that determines the parameters of wireless channel by reducing the sum of the squared differences between the predicted channel and the received signals based on linear equations without requiring the statistical properties of the channel. LS are often applied in Pilot-based channel estimation to achieve optimal performance at a reasonable computational complexity but deteriorate in performance at low SNR because it does not capture the effect of noise [61].
- ii. **Minimum Mean Square Error:** MMSE estimation algorithm is a powerful technique for MIMO channel parameter estimation due to its robustness to noise and channel variability. It performs more optimally than LS but more complex than LS which requires no statistical information or matrix inversion [62]. It strikes a good balance between estimation accuracy and computational intensity, leading to its wide applications in real-time MIMO communication systems. It requires the statistical information of the wireless channel to achieve a minimal MSE and BER than LS.
- iii. **Linear Minimum Mean Square Error:** LMMSE is an optimal linear estimator that offers minimum mean square error with increased computational burden due to reliance on the knowledge of the channel and noise statistics. It is suitable for real-time implementations in a pilot-based channel estimation technique to provide more reliable channel estimation in noisy and fading channels.
- iv. **Compressed Sensing:** Compressive Sensing is a three-step innovative algorithm comprising of representation, sampling and reconstruction stages. The application of CS enables accurate signal detection from sparse or compressible measurements even below the Nyquist rate [63]. Examples of Compressive Sensing Algorithms include Basis Pursuit (BP), Orthogonal Matching Pursuit (OMP), Compressive Sampling Matching Pursuit (CoSaMP).
- v. **Expectation-Maximization:** EM algorithm is an efficient two step iterative approach for carrying out maximum likelihood estimation when some variables are unknown or hidden. The two-step iterative procedures include repeated estimation of the unknown parameters and optimization of the model until the value converges to a local maximum. To avoid suboptimal convergence, initialization should be definite. EM algorithm is majorly employed in semi-blind channel estimation technique especially when dealing with scenarios with incomplete data.
- vi. **Kalman Filter:** KF is a recursive algorithm that estimates the state of a dynamic system such as wireless channels using noisy measurements. It combines the priori estimates and fresh measurements to achieve optimal estimation. In the KF algorithm, the tracking procedure is in two stages: the prediction stage and the update stage which is made up of prediction equations and correction or update equations respectively. The prediction equation predicts the state of the system using a selected model while the correction equation addresses errors and inaccuracies that appear during the estimation process.
- vii. **Maximum Likelihood:** The ML algorithm utilizes the likelihood function to estimate wireless channel parameters. In this approach, the algorithm estimates the parameters of the channel that maximizes the likelihood of the estimated parameter with the assumption that the noise statistics is Gaussian. In pilot-based channel estimation technique, the received pilot is used to compute the likelihood function and achieve optimal channel parameters. However, it computationally and spectrally inefficient as it requires much pilot overhead.

Table 2: Performance evaluation of improved MIMO channel estimation techniques

Citation	Basic Algorithm	Metrics						
		SNR	MSE	NMSE	BER	SDR	SER	Accuracy
[21]	LS	✓			✓			
[23]	MMSE	✓		✓				
[22]	LMMSE	✓			✓			
[26]	Square root	✓	✓					
[27]	Correntropy ICA	✓	✓			✓		
[28]	SOBI	✓	✓			✓		
[29]	Kalman Filter	✓		✓	✓			
[30]	robust KF	✓			✓			
[35]	Linear prediction and LS	✓	✓		✓			
[36]	Forward-only Kalman filter	✓			✓			
[38]	Classical Basis Pursuit	✓	✓				✓	
[39]	CoSaMP	✓	✓					
[42]	CNN	✓		✓				✓



[43]	NN and CNN	✓		✓				
[44]	(DRN)-based MMSE	✓		✓				✓
[64]	CRLD	✓		✓				✓
[65]	LS and MMSE	✓		✓				

5.0 CONCLUSION

Channel estimation is crucial for exploiting the inherent advantages of MIMO technology in wireless communication systems. We have discussed various channel estimation techniques and their performances using key performance metrics, including the challenges of outdated CSI in channel estimation. Several channel estimation algorithms are discussed as well as the application of Kalman filter and its variants in channel estimation, pointing out the limitations and the improved performances of each variant. Moreover, Neural Network-based channel estimation techniques promises a more robust and accurate estimation for various channel models. High complexity and dynamic nature of wireless channels which leads to Frequent expiration of CSI are key challenges in MIMO channel estimation, we hence recommends the integration of recent machine learning techniques to handle the challenge of complexity; and also a hybrid technique that will incorporate conventional techniques with advanced algorithms that can adapt to variations in user resource demands and channel conditions.

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