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MATHEMATICAL MODELLING OF COVID-19 TRANSMISSION DYNAMICS IN NIGERIA WITH CONTROL THROUGH VACCINATION, TREATMENT, AND AWARENESS CAMPAIGNS

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Abstract

The COVID-19 pandemic, though significantly reduced by 2023, still poses a risk of resurgence in Nigeria due to suboptimal vaccination uptake and weak adherence to preventive measures. To better understand and strengthen control strategies, this study develops a mathematical model of COVID-19 transmission dynamics with control through awareness campaigns, use of safe vaccines, and treatment of infected individuals. The proposed model is formed using a system of nonlinear ordinary differential equations with eight compartments. Analytical results established the boundedness and positivity of the solution, as well as the existence and stability conditions for both the disease-free and endemic equilibria. The basic reproduction number, derived using the next-generation matrix approach, was greater than one in the absence of interventions but declined below unity when vaccination and awareness coverage improved, indicating potential elimination of the disease. Sensitivity analysis revealed that vaccination completion rate, isolation with treatment, and awareness campaigns are the most influential parameters in reducing transmission. Numerical simulations, based on Nigerian data, showed that without awareness campaigns COVID-19 could persist with high infection levels, while increased vaccination awareness markedly reduced susceptible and infectious populations. Long-term projections further indicated that sustained vaccination, awareness campaigns, and treatment can eradicate the disease. The results of the analysis showed that the proposed controls are effective to eradicate and prevent a resurgence of COVID-19 in Nigeria. Hence, it is recommended that public health awareness, vaccination, and treatment be implemented at full scale for effective disease control.

1.0 INTRODUCTION

The illness caused by the novel coronavirus SARS-CoV-2 was officially designated as Coronavirus Disease 2019 (COVID-19) by the World Health Organisation (WHO) [1]. Following a report of a cluster of instances of "viral pneumonia" in Wuhan, People's Republic of China, the World Health Organisation (WHO) first became aware of this virus on 31st December, 2019 [1]. The WHO article described Coronavirus Disease (COVID-19) as an infectious disease that was first diagnosed in the Chinese city of Wuhan in December 2019 and is brought on by a coronavirus code-named COVID-19 by WHO [2].

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This work responds to WHO Situation Report 90, which highlighted the need to utilise networks of researchers and specialists to coordinate international efforts in surveillance, epidemiology, and mathematical modelling, among other responses to COVID-19. To this end, it is imperative to mathematically model the novel COVID-19 pandemic to aid containment of the spread, diagnostics, clinical care, treatment and infection prevention, among others [3]. There has been advocacy for the use of preventive measures to control the spread of the disease, including isolation, quarantine, enlightenment for and use of safe vaccines (see [1] and [4]), which the model put into consideration.

As of 26 February 2023, Nigeria recorded 5,915,616 laboratory tests, with 266,313 confirmed cases, 259,027 recoveries, 4,131 active cases, and 3,155 COVID-19-related deaths [5]. According to WHO, there have been *770,085,713* confirmed cases of COVID-19 and *6,956,173* COVID-19-induced deaths as of 12:20 pm CEST, 30th August 2023. A total of 13,499,983,736 vaccine doses have been administered by 27 August 2023 worldwide [6].

A review of previous studies has showed several studies on the disease in a Nigerian setting. See [7], [8], [9] and [10]. This work, unlike previous studies, considered the control of COVID-19 spread using WHO and NCDC statistics from August 2023. It aimed at achieving complete eradication and preventing resurgence of the disease in Nigeria, despite poor public attitudes towards vaccination, in line with WHO and NCDC recommendations and global best practices for pandemic prevention and control [6]. The study aimed to develop a model of COVID-19 transmission dynamics with control measures including quarantine, isolation, vaccination and treatment. Specifically, the objectives of the model are to: (a) develop a model of COVID-19 transmission dynamics to inform effective control strategies; (b) establish the invariant region and positivity of solutions to ensure the model realistically represents human populations; (c) obtain the steady states of the model and determine the conditions under which the disease can persist or die out; (d) estimate the basic reproduction number as an indicator of potential spread and assess conditions for control or eradication; (e) identify the most influential drivers of transmission through sensitivity analysis and provide guidance for effective public health interventions.

2.0 MATERIALS AND METHODS

2.1 Model of COVID- Ransmission Formulation

The study puts into consideration the natural history of COVID-19 transmission from the medical literature and presents the mathematical model related to the pandemic's pattern of transmission, from person-to-person transmission. We formulated an eight-compartment model of COVID-19 transmission, incorporating control measures such as vaccination, quarantine, isolation with treatment, and health education through awareness campaigns. Precisely: S = susceptible, V = vaccinated, V_C= complete vaccination, E = exposed. Also, other population compartments include: Q = quarantined persons, I = infectious persons (COVID-19 case): I_T= isolation in public health facilities with treatment and R = recovered.

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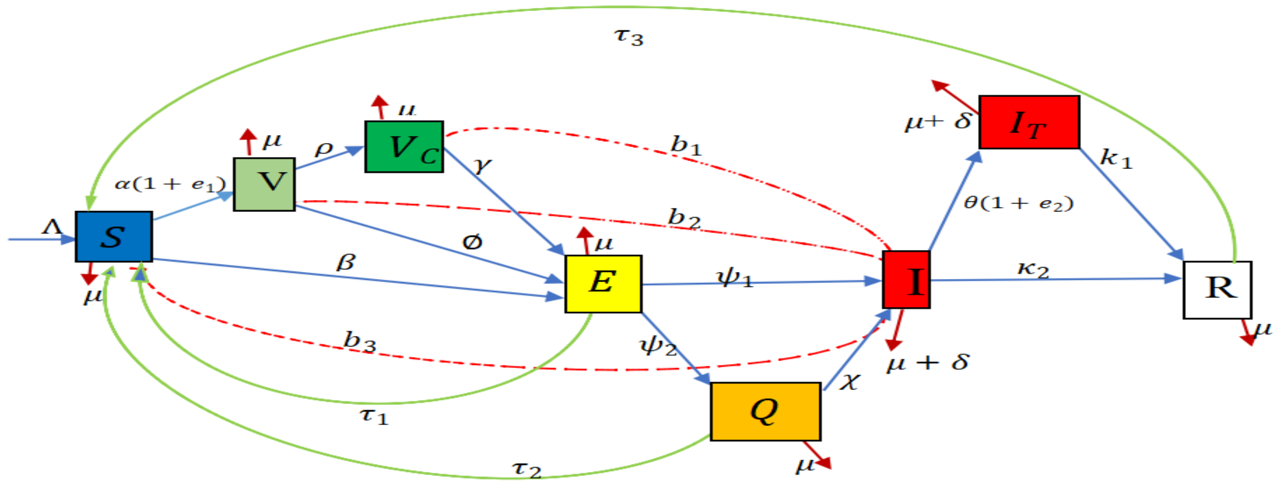

Figure 1: COVID-19 transmission dynamics

Model Major Assumptions

- Susceptible individuals can only get infected after contact with an infectious person(s) or environment (infectious surfaces or objects).
- Age, sex, social status, and race are assumed not to affect the probability of COVID-19 infection.
- Individuals who recover from COVID-19 are assumed not to acquire lasting immunity.
- Variants of COVID-19 may exist against which current vaccines do not provide immunity.
- Exposed individuals to COVID-19 that did not get the infection return to susceptible classes.
- There is no inherited immunity against COVID-19; therefore, every human without vaccination is susceptible to COVID-19.
- There is the situation of infectious individuals to be strictly isolated in public health facilities called government-approved isolation centres.
- There are chances for natural recovery from COVID-19, and the vaccines can wane over time.
- Isolated individuals are assumed not to infect susceptible persons due to strict containment measures.

2.2 The Model of Covid-19 Transition Dynamics

From Figure 1, the blue arrow is the movement of persons from one compartment to another, the green arrow is movement of persons back to the susceptible class and the dotted curves are the interaction of persons with infectious persons. The mechanistic model of COVID-19 pandemic dynamics presented in the following eight compartment classes of differential equations that are time dependent; (1),(2) ,(3) …(8).

$$\dot{S} = \Lambda - [\alpha(1+e_1) + b_3 I + \mu]S + \tau_1 E + \tau_2 Q + \tau_3 R \quad (1)$$

$$\dot{V} = \alpha(1+e_1)S - (\rho + b_2 I + \mu)V \quad (2)$$

$$\dot{V_C} = \rho V - (b_1 I + \mu)V_C \quad (3)$$

$$\dot{E} = b_3 IS + b_2 IV + b_1 IV_C - (\psi_1 + \psi_2 + \mu + \tau_1)E \quad (4)$$

$$\dot{Q} = \psi_2 E - (\chi + \mu + \tau_2)Q \quad (5)$$

$$\dot{I} = \psi_1 E + \chi Q - [\delta + k_2 + \mu + \theta(1+e_2)]I \quad (6)$$

$$\dot{I_T} = \theta(1+e_2)I - (\delta + k_1 + \mu)I_T \quad (7)$$

$$\dot{R} = k_1 I_T + k_2 I - (\mu + \tau_3)R \quad (8)$$

Table 1: Model epidemiological definitions

Variable	Biological Interpretation
S	Susceptible: Population that can be exposed to or infected by COVID-19.
V	Vaccinated; vaccinated population among the susceptible class.

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V_C	Complete Vaccination: individuals who received the full COVID-19 vaccination.
E	Exposed: individuals who had contact with COVID-19-infectious persons, objects or environments (probable cases).
Q	Quarantined Persons: these are individuals with contact with the COVID-19 virus but removed from other members of the susceptible population.
I	Infectious Persons (COVID-19 Cases): individuals infected with COVID-19 include symptomatic, asymptomatic or pre-symptomatic individuals.
I_T	Isolated persons undergoing treatment: Infectious individuals isolated and receiving COVID-19 treatment.
R	Recovered: Infectious individuals that have fully recovered from COVID-19.

Table 2: Model parameters definitions

Param.	Biological Interpretation	Value	Source
Λ	Λ is the recruitment level into COVID-19-susceptible population.	750	[8]
α	The rate at which the susceptible population took vaccination against COVID-19 in Nigeria.	0.6233	[6]
ρ	The rate of completion of vaccination among vaccinated persons.	0.06615	Estimated from [6]
ψ_1	The rate at which persons exposed to COVID-19 get infected by the virus.	0.04671	Estimated from [6]
ψ_2	The rate at which persons exposed to COVID-19 get quarantined.	0.75	Estimated from [5]
e_1	The rate at which awareness makes susceptible persons go for COVID-19 vaccination.	0.5	Assumed (placeholder)
e_2	The rate at which awareness of the dangers of COVID-19 infection makes an infectious person accept isolation with appropriate treatment.	0.5	Assumed (placeholder)
χ	The rate at which quarantined individuals who became infectious with COVID-19 move into the infectious class.	0.75	Estimated from [5]
τ_1	The rate at which persons exposed to COVID-19, who were neither quarantined nor COVID-19 infectious, return to the susceptible class.	0.6	Assumed
τ_2	The rate at which persons exposed to COVID-19, who were quarantined but not infected by COVID-19 return to the susceptible class.	0.5	Estimated from [5]
τ_3	The rate at which recovered individuals from COVID-19, return to the susceptible class.	0.97479	Estimated from [5]
θ	The rate at which infectious COVID-19 individuals accept to go for isolation with treatment.	0.25	Assumed
κ_1	The rate of isolated infectious individuals with COVID-19 undergoing treatment recover from COVID-19.	0.97479	Estimated from [5]
κ_2	The rate of COVID-19 infectious persons without isolation or treatment gets recovered from COVID-19.	0.67	Assumed

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μ	Natural death (non-COVID-19-induced mortality) rate among humans.	0.0000432	
δ	The rate of COVID-19 induced death.	0.01184	[5]
b_1	The rate at which susceptible persons with complete vaccination interact with infectious persons.	0.000000001	Assumed
b_2	The rate at which vaccinated persons interact with infectious persons.	0.001 & 0.02	Assumed
b_3	The rate at which susceptible persons without vaccination interact with infectious persons.	0.0001	Assumed

2.3 Invariant Region and Positivity Analysis

The invariant region refers to the set (space, surface, or curve) in which a system of differential equations, or its solution, begins and remains unchanged. Given the invariant set, also referred to as manifold that is positive, the solution of the differential equation in the positive region will be positive at all times. Hence, a positively invariant set or manifold, will yield solutions that remain positive for all time. Since in our model, we are studying the human population, all solutions of the equations (1), (2), (3) …(8) are positive for every $t > 0$ if they enter the invariant region.

$$\Omega = \left\{(S,V,V_C,E,Q,I,I_T,R) \in \mathbb{R}_+^8 : N < \frac{\Lambda}{\mu}\right\} \qquad (9)$$

This was established in line with [11], [7], [8] and [12]. Hence, we proved that the model state variables are positive in the invariant region Ω.

Proof: The model's solution with non-negative initial conditions is denoted by closed set Ω. At $t>0$, our model of human population compartments all states variables are semi-positive definite.

$S \geq 0, V \geq 0, V_C \geq 0, E \geq 0, Q \geq 0, I \geq 0, I_T \geq 0, R \geq 0$

. The entire population is given by

$$N = S + V + V_C + E + Q + I + I_T + R \qquad (10)$$

This implies $\frac{dN}{dt} = \frac{dS}{dt} + \frac{dV}{dt} + \frac{dV_C}{dt} + \frac{dE}{dt} + \frac{dQ}{dt} + \frac{dI}{dt} + \frac{dI_T}{dt} + \frac{dR}{dt}$, summation of (1),(2), (3) ….(8).

Hence, $\frac{dN}{dt} + \mu N \leq \Lambda$ (11).

Comparing the ODE over time: $\frac{dx}{dt} + P(t)x = Q(t)$ (12)

with integrating factor: $IF = e^{\mu t}$. Solution showed $N(t) \leq \frac{\Lambda}{\mu} + \left\{N(0) - \frac{\Lambda}{\mu}\right\} e^{-\mu t}$. As $t \to \infty$, $N(0) \leq \frac{\Lambda}{\mu}$, then $N(t) \leq \frac{\Lambda}{\mu}$. Therefore,

$\Omega = \left\{(S,V,V_C,E,Q,I,I_T,R) \in \mathbb{R}_+^8 : N < \frac{\Lambda}{\mu}\right\}$ proved.

The manifold is a positive variant. Hence, the solution to the system is bounded in Ω which indicates the epidemiology model is well-posed in its mathematical sense [10] and [13].

On positivity of the solution, we solved equation (1) of the model;

$$\frac{dS}{dt} = \Lambda - [\alpha(1 + e_1) + b_3 I + \mu]S + \tau_1 E + \tau_2 Q + \tau_3 R$$

.

This implies $\frac{dS}{dt} + [\alpha(1 + e_1) + b_3 I + \mu]S \geq \Lambda$ (13).

Compared (13) and (12), the integrating factor $IF = e^{\mu t + \int_0^t [(1+e_1)\alpha(\xi) + b_3 I(\xi)]d\xi}$.

At initial condition, $t = 0$, $S(0) \geq c_1$. Therefore, the solution;

$$S(t) \geq \left\{\int_0^t \Lambda e^{\mu y + \int_0^y [(1+e_1)\alpha(\xi) + b_3 I(\xi)]d\xi} dy\right\} e^{-\{\mu t + \int_0^t [(1+e_1)\alpha(\xi) + b_3 I(\xi)]d\xi\}} + S(0) e^{-\{\mu t + \int_0^t [(1+e_1)\alpha(\xi) + b_3 I(\xi)]d\xi\}} > 0.$$

This showed $S(t) > 0$. Similarly, solution of equation (2) of the model, implies the solution

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$$V_C(t) \geq V_C(0)e^{-\mu t - \int_0^t b_1 I(\xi)d\xi} > 0.$$

Hence, $V_C(t) > 0$. The solution of other equations by separation of the variable method showed that; $E(t) > 0$, $Q(t) > 0$, $I(t) > 0$, $I_T(t) > 0$, and $R(t) > 0$.

2.4 The Steady States of the Model

The model's disease-free equilibrium (DFE) represents the state where no infection exists in the population. Establishing the existence of the DFE means demonstrating mathematically that the system admits such an equilibrium point under specified conditions. At DFE, the state variables of the disease compartments

$$E = Q = I = I_T = R = 0,$$
$$\dot{S} = \dot{V} = \dot{V}_C = \dot{E} = \dot{Q} = \dot{I} = \dot{I}_T = \dot{R} = 0$$

Therefore, from equations (1) , (2) …(8);

$$(S^0, V^0, V_C^0, E^0, Q^0, I^0, I_T^0, R^0) = \left(\frac{\Lambda}{\alpha+\alpha e_1+\mu}, \frac{\alpha\Lambda(1+e_1)}{(\rho+\mu)(\alpha e_1+\alpha+\mu)}, \frac{\alpha\rho\Lambda(1+e_1)}{(\alpha+\alpha e_1+\mu)(\rho+\mu)\mu}, 0,0,0,0,0\right) \quad (14)$$

On the Existence of Endemic Equilibrium Point (EEP);

$$(S, V, V_C, E, Q, I, I_T, R) = (S^*, V^*, V_C^*, E^*, Q^*, I^*, I_T^*, R^*).$$

From (7).

$$I_T^* = \frac{\theta(1+e_2)I^*}{\delta+k_1+\mu} \quad (15)$$

$$R^* = mI^*, \text{ where } m = \frac{(\theta e_2 k_1 + \delta k_2 + \mu k_2 + \theta k_1 + k_1 k_2)}{(\delta+k_1+\mu)(\mu+\tau_3)} \quad (16).$$

$$Q^* = nI^*, \text{ where } n = \frac{\psi_2(\theta e_2+\delta+\mu+\theta+k_2)}{\chi\psi_1+\chi\psi_2+\mu\psi_1+\psi_1\tau_2} \quad (17).$$

$$E^* = \frac{n(\chi+\mu+\tau_2)}{\psi_2} I^* \quad (18).$$

$$S^* = \frac{\Lambda\psi_2+\tau_1 n(\chi+\mu+\tau_2)I^*+\tau_2\psi_2 nI^*+\tau_3\psi_2 mI^*}{\psi_2(\alpha+\alpha e_1+\mu+b_3 I^*)} \quad (19).$$

$$V^* = \frac{(\alpha+\alpha e_1)[\Lambda\psi_2+\tau_1 n(\chi+\mu+\tau_2)I^*+\tau_2\psi_2 nI^*+\tau_3\psi_2 mI^*]}{\psi_2(\alpha+\alpha e_1+\mu+b_3 I^*)(\rho+\mu+b_2 I^*)} \quad (20).$$

$$V_C^* = \frac{\rho(\alpha+\alpha e_1)[\Lambda\psi_2+\tau_1 n(\chi+\mu+\tau_2)I^*+\tau_2\psi_2 nI^*+\tau_3\psi_2 mI^*]}{\psi_2(\mu+b_1 I^*)(\alpha+\alpha e_1+\mu+b_3 I^*)(\rho+\mu+b_2 I^*)} \quad (21).$$

Hence, there exists EEP for the model.

2.5 COVID-19's Basic Reproduction Number

The reproduction number $\boldsymbol{R_0}$ of an infection is the critical threshold which indicates the total number of fresh infections brought on by one illness case [14]. We computed the $\boldsymbol{R_0}$ using Next generation method approach of [15] because the system contains multiple infectious compartments. Our disease compartment are; equation (4) to equation (7). We obtain the following:

$$f_i = \begin{pmatrix} b_3 SI + b_2 IV + b_1 IV_C \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$v_i = \begin{pmatrix} (\psi_1+\psi_2+\mu+\tau_1)E \\ -\psi_2 E + (\chi+\mu+\tau_2)Q \\ -\psi_1 E - \chi Q + [\delta+k_2+\mu+\theta(1+e_2)]I \\ -\theta(1+e_2)I + (\delta+k_1+\mu)I_T \end{pmatrix}$$

$$FV^{-1} = \begin{pmatrix} \frac{(b_3S+b_2V+b_1V_C)(\chi\psi_1+\chi\psi_2+\mu\psi_1+\psi_1\tau_2)}{(\chi+\mu+\tau_2)(\psi_1+\psi_2+\mu+\tau_1)[\delta+k_2+\mu+\theta(1+e_2)]} & \frac{(b_3S+b_2V+b_1V_C)\chi}{(\chi+\mu+\tau_2)[\delta+k_2+\mu+\theta(1+e_2)]} & \frac{b_3S+b_2V+b_1V_C}{\delta+k_2+\mu+\theta(1+e_2)} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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R_0 is the greatest eigen value of the matrix FV^{-1}, obtained with the help of Maple 18. When evaluated at the DFE (14), we obtained:

$$R_0 = \frac{\Lambda(\alpha\mu b_2 e_1 + \alpha\rho b_1 e_1 + \alpha\mu b_2 + \alpha\rho b_1 + \mu^2 b_3 + \mu\rho b_3)(\chi\psi_1 + \chi\psi_2 + \mu\psi_1 + \psi_1\tau_2)}{\mu(\alpha e_1 + \alpha + \mu)(\rho + \mu)(\chi + \mu + \tau_2)(\psi_1 + \psi_2 + \mu + \tau_1)[\delta + k_2 + \mu + \theta(1 + e_2)]}$$

.

2.6 Establishing the Condition for Stability

The stability of the steady states helps to determine how long COVID-19 persists in the population. The stability of DFE determines the short-term dynamics of an infectious disease, while endemic equilibrium provides the long-term dynamics.

Theorem 2.7.1: A system is locally and asymptotically stable at a point if the real part of all the eigenvalues is negative at the point.

Theorem 2.7.2: The DFE is locally and asymptotically stable if $R_0 < 1$and unstable if $R_0 > 1$. [15]. We established the local stability of DFE using linearization method [16]. The Jacobian of the model is given by

$$J^0 = \begin{pmatrix} -(\alpha + \alpha e_1 + \mu) & 0 & 0 & \tau_1 & 0 & 0 & 0 & 0 \\ \alpha(1 + e_1) & -(\rho + \mu) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho & -\mu & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(\mu + \tau_1) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -[\mu + \theta(1 + e_1)] & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \theta(1 + e_2) & -\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu \end{pmatrix}$$

The model's Jacobian matrix's eigenvalues at DFE generate:

$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = -\mu, \lambda_5 = -\rho - \mu, \lambda_6 = -\alpha e_1 - \alpha - \mu, \lambda_7 = -[\mu + \theta(1 + e_1)]$

and

$\lambda_8 = -(\mu + \tau_1)$. This indicated that in the absence of the disease, the eigenvalues are real and all negative. That is $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8 < 0$. This entails that the system is local and asymptotically stable. Also, the DFE is locally and asymptotically stable if $R_0 < 1$ by Theorem 2.7.2. This entails that the COVID-19 will be eliminated in the human population given certain conditions because when the average infected persons as a result of one infected person is less than one, a small presence of COVID-19 infectious individuals into the society will not generate a large COVID-19 outbreak. With appropriate control measures the disease could be eradicated.

The global stability of the equilibrium point followed the method of [16] use of LaSalle's invariant principle to construct Lyapunov function. The Lyapunov function is positive definite and its first-time derivative is semi-negative definite.

Proof: let Z be the Lyapunov function.

$$Z = S - S^0 - S^0 \ln\frac{S}{S^0} + V - V^0 - V^0 \ln\frac{V}{V^0} + V_C - V_C^0 - V_C^0 \ln\frac{V_C}{V_C^0} + E - E^0 - E^0 \ln\frac{E}{E^0} + Q - Q^0 - Q^0 \ln\frac{Q}{Q^0} + I - I^0 - I^0 \ln\frac{I}{I^0} + I_T - I_T^0 - I_T^0 \ln\frac{I_T}{I_T^0} + R - R^0 - R^0 \ln\frac{R}{R^0} \quad (22)$$

At DFE, the first-time derivative of the function is given by;

$\dot{Z} = \dot{S} - \frac{S^0}{S}\dot{S} + \dot{V} - \frac{V^0}{V}\dot{V} + \dot{V}_C - \frac{V_C^0}{V_C}\dot{V}_C$ since, $E^0 = Q^0 = I^0 = I_T^0 = R^0 = 0$ substituting (1) to (3)

$\therefore \dot{Z} = M - N$. The first-time derivative of the Lyapunov function is Semi-negative definite if $N > M$. This entails the DFE is global and asymptotically stable if $N > M$.

On global stability of endemic equilibrium Point (EEP), going by Theorem 2.7.2 the EEP of our COVID-19 model is globally asymptotically stable given that our model basic reproduction number; $R_0 > 1$.

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3.0 RESULTS AND DISCUSSIONS

3.1 Computation and Discussion of R_0

In line with Theorem 2.6.3 which entailed that when the infections cause by a single infectious person within the environment is greater than one, our endemic equilibrium point will be stable in the defined human population but unstable when it is less than one while Theorem 2.7.2 states that the DFE will be stable when the reproduction number is less than one. This indicates that if the number of new infections generated in our environment by a single infectious individual is more than one, the persistence of COVID-19 in our environment will last for a long time except certain control measures are applied to curtail the existence of the deadly disease. COVID-19 reproduction number of the model using the NCDC statistics, computed for $0 < e_1 < 1$ and $0 < e_2 < 1$ the enlightenment rate for vaccination and isolation with treatment; R_0 values were given by R_0= [9.48720172 9.48629515 9.48542412 ... 7.46254508], R_0= [0.52434079 0.52342178 0.52253879 ... 0.39531315], R_0= [0.09978422 0.09886462 0.09798107 ... 0.06054953]; at rate of interaction of the of vaccinated persons with infectious person given by $b_2 = 0.02$, 0.001, 0.001 and 0.0001respectively.

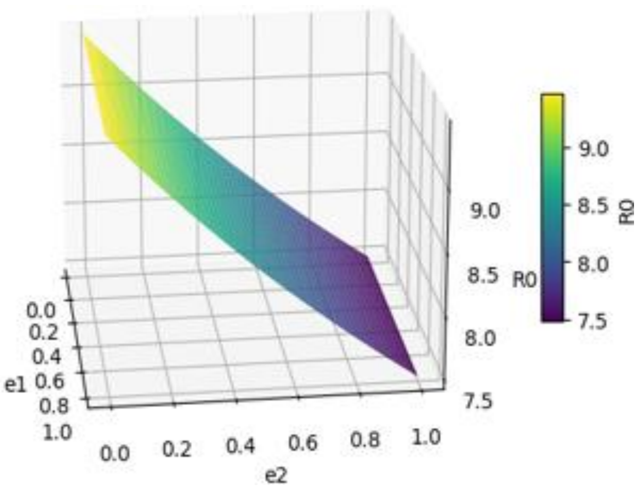

Figure 2a: Variation of the $\boldsymbol{R_0}$ with awareness and contact rate at 0.2

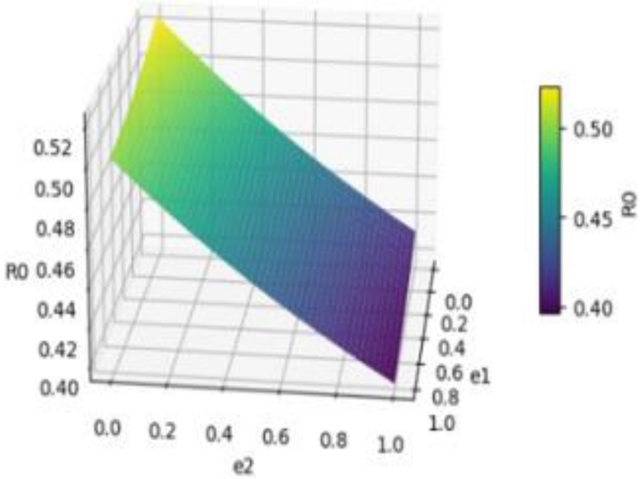

Figure 2b: Variation of the $\boldsymbol{R_0}$ with awareness and contact rate at 0.001

Figure 2 is the 3D python plot of for b_2 =0.02, 0.001, 0.0001, this result indicates that the rate at which an infectious person can generate new cases can be as high as 9.47 when rate of interaction with vaccinated persons is 2% amidst population with COVID-19 infection. This is alarming because of high transmission rate. On the positive note, with enlightenment for use of vaccine and isolation of infected persons with treatment at 100% when possibility of infection among vaccinated persons is at 0.001, the rate at which an infectious person can generate new infections is 0.3953. this can be as low as 0.06055 when the interaction with infectious persons is lowered to 0.0001 which by theorem 2.7.2, COVID-19 will fizzle out from Nigerian population in a short time if these conditions are met.

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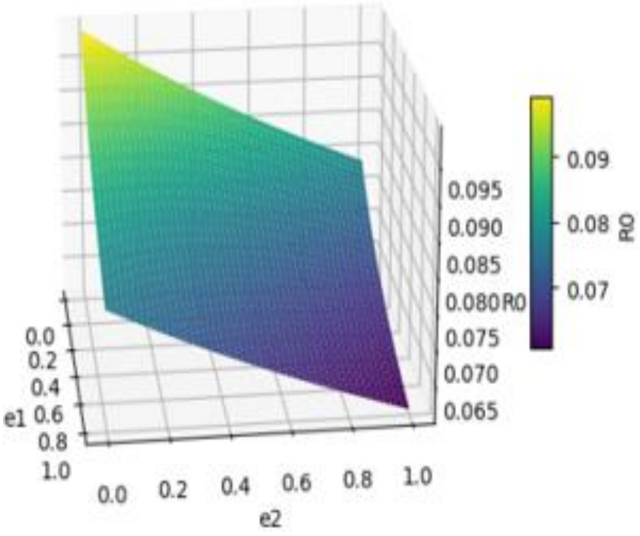

Figure 2c: Variation of the $\boldsymbol{R_0}$ with awareness and contact rate at 0.0001

3.2 Model Sensitivity Analysis

In need to obtain the optimal control of the spread of COVID-19, the study foud out the parameters to focus on to achieve the best intervention strategies to control the spread and prevent resurgence of COVID-19 within Nigerian setting. The parameters' sensitivity indices were computed for all parameter using Python code. See Table 3.

Figure 3 is the python plot of Table 3, the parameters with positive values as represented with green bars indicate parameters that their increase show positive increase in R_0. This implies that the increase in such parameters result to increase in the transmission of COVID-19. Red bars present the parameters that must be increased to ensured reduction in the spread. As the parameters with negative index are reduce, this facilitates the COVID-19 virus's spread.

Table 3: Sensitivity indices of the parameters

Param.	Indices	Param.	Indices	Param.	Indices	Param.	Indices	Param.	Indices
Λ	1.0000	ψ_2	0.3690	τ_1	-0.4296	κ_1	0	b_1	0.0007
α	-0.0032	e_1	-0.0011	τ_2	-0.3624	κ_2	-0.6339	b_2	0.9961
ρ	-0.9960	e_2	-0.1183	τ_3	0	μ	-0.0009	b_3	0.0032
ψ_1	0.0606	χ	0.3624	θ	-0.3548	δ	-0.0112		

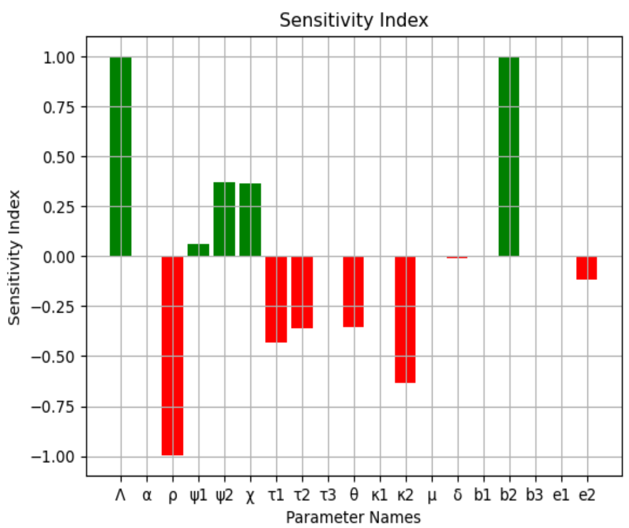

Figure 3: Significance of parameters in $\boldsymbol{R_0}$

This result shows that Λ and b_2 are the two most influential parameters that must be reduced to guarantee a decrease in the transmission of Covid-19, followed by e_2 and χ among others. So, the movement of persons into the given population in any setting in Nigeria amidst of COVID-19 outbreak must be check to avoid the spread of the disease. Also, the rate at which persons without complete vaccination have contact with COVID-19 infectious persons must be reduced to check the spread of the disease. Restriction on social gathering and use of protective measures like face mask, social distancing among others is ideal. On the negative indices; ρ, κ_2, τ_1, τ_2 and θ, show parameters that their values require increment to lower the COVID-19 transmission rate in Nigeria. This entails that there is need to increase the rate of completion of vaccination among the people, the rate at which COVID-19 infectious person isolate and present themselves for treatment and to enhance natural recovery rate among infectious persons.

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3.3 Simulation of the Model Compartments and Results

Using the parameter values on Table 2, the model was simulated using the initial condition of the state variables [11]. When the initial susceptible population at the time of introduction of the COVID-19 in a location was $S(0) = 5000$, within a location in Lagos, Nigeria obtained from [8], where $V(0) = 1000$ has taken first jab of the vaccine, $V_C(0) = 300$ completed vaccination, $E(0) = 1000$ already exposed to the disease, $Q(0) = 600$ quarantined, $I(0) = 1000$ infected, $I_T(0) = 60$ isolated and undergoing treatment and $R(0) = 10$ recorded recovery.

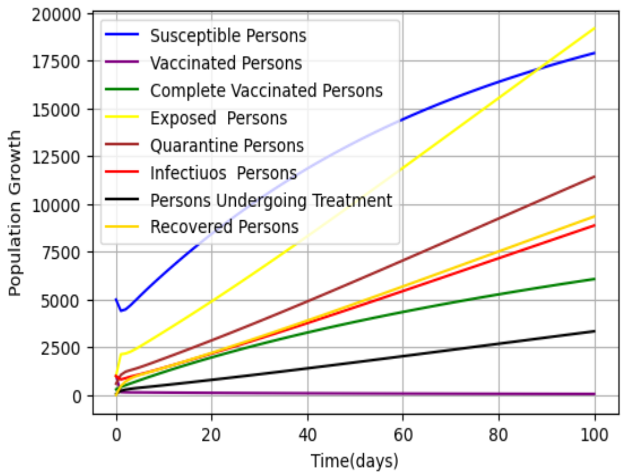

Figure 4: COVID - 19 model compartments with e1 = 0 and b2= 0.02 (100 days without vaccine enlightenment)

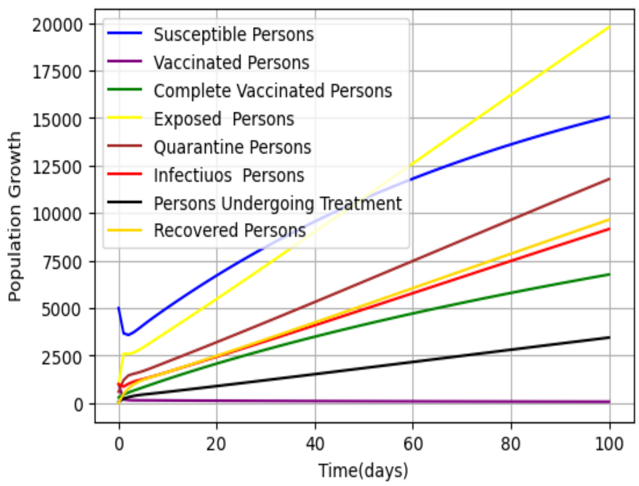

Figure 5: COVID - 19 model compartments with e1 = 0.5 and b2= 0.02 (100 Days with vaccine enlightenment at 50%)

Figure 4 from Python plot of the model compartments shows that without awareness/enlightenment for vaccination, at a contact rate of persons with incomplete vaccination with COVID-19 at 0.02, the result indicated that within 100 days of COVID-19 introduction into the initial population of 5000 susceptible persons in a setting, the disease can be found in large population with suceptible population above 17500 persons, exposed persons (contacts) to the disease at 19000 with possible cases of 11000 Quarantined persons, over 8000 cases of infections, over 6000 persons completed vaccination doses, possible 6000 recovered persons and about 3000 isolated persons undergoing treatment. Comparing 50% enlightenment of Nigerians for use of vaccines as shown on Figure 5, the result indicated reduction of susceptible persons from 17500 down to 15000 and increment of persons with complete doses of vaccine from 6000 to 7000 persons.

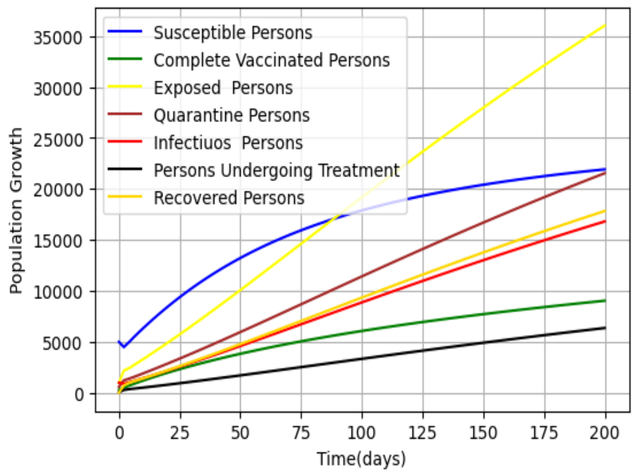

Figure 6: COVID - 19 model compartments with e1 = 0 and b2= 0.02 (200 Days without vaccine enlightenment)

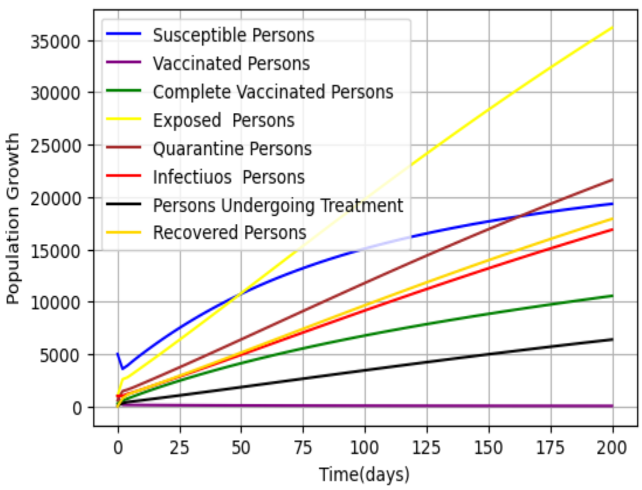

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Figure 7: COVID - 19 model compartments with e1 = 0.5 and b2= 0.02 (200 Days with vaccine enlightenment at 50%)

Using the same values of the parameters and initial population in 200 days, Python plot in Figure 6 indicated that persons with contact to infected inidividual will be as high as 35000 persons, with suceptible and recovery cases approximately 20000 each. The figure also showed that there will be almost 15000 cases of infections, less than 10000 persons that completed vaccination doses, possible 22000 recoveries and 2500 isolated persons undergoing treatment. While on Figure 7 with enlightenment for susceptible persons to go for vaccination, the population of susceptible persons reduced below 20000 persons with individuals that has received complete vaccination rises above 10000 persons.

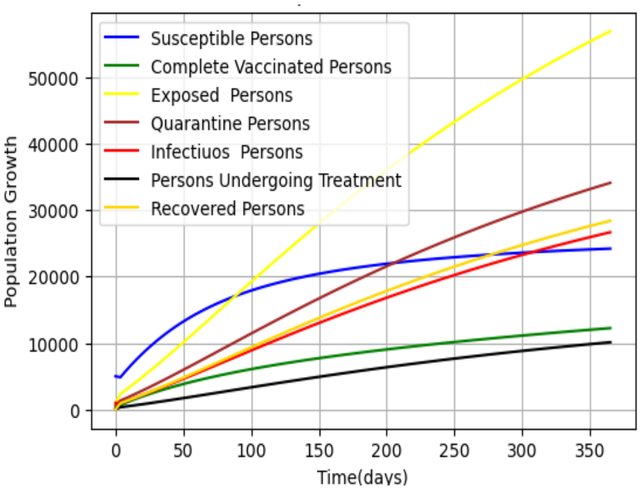

Figure 8: COVID - 19 model compartments with e1 = 0 and b2= 0.02 (1 Year without vaccine enlightenment)

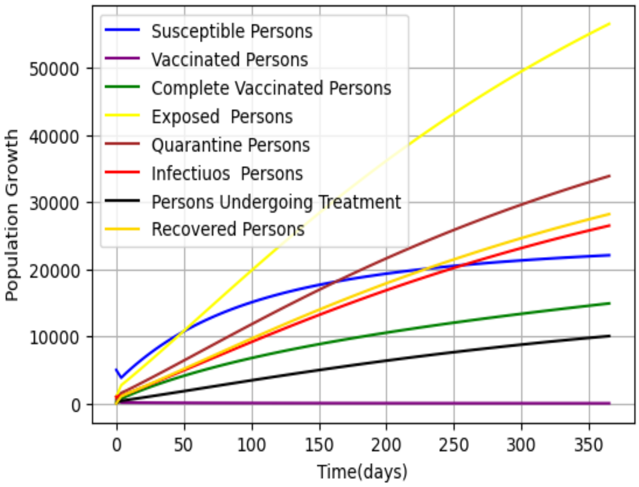

Figure 9: COVID - 19 model compartments with e1 = 0.5 and b2= 0.02 (1 Year with vaccine enlightenment at 50%)

Figure 8 and Figure 9 entail that at 0.02 rate of interaction of persons with COVID-19 the population of persons with complete vaccination is approximately 11000 in one year without enlightenment. The value rises to 15000 with 50% enlightenment for vaccination. The suscptible persons in one year time decreased from 24000 to 21000 when the enlightenment for vaccination was placed at 50%. This result supports the need for enlightenment for use of safe vaccines to control COVID-19 spread.

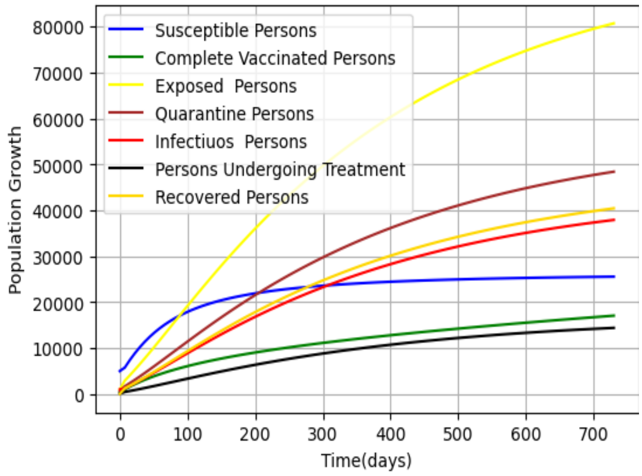

Figure 10: COVID - 19 model compartments with e1 = 0 and b2= 0.02 (2 Years without vaccine enlightenment)

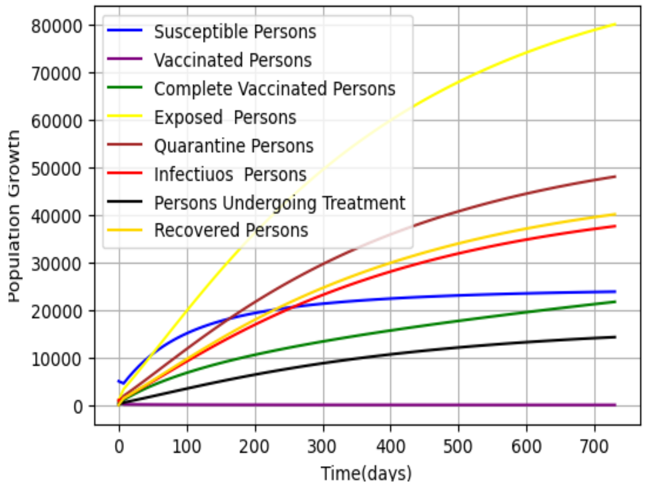

Figure 11: COVID - 19 model compartments with e1 = 0.5 and b2= 0.02 **(**2 Years with vaccine enlightenment at 50%)

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In two years time, Figure 10 without enlightenment for vaccination projected exposed persons from the initial population in the setting at 80000, almost 50000 quarantined, 39000 cases of COVID-19, 40000 recovered cases with 26000 Susceptible persons. While isolation with treatment is 28000 persons. While on Figure 11 with enlightenment for susceptible persons to go for vaccination, the population of susceptible persons reduced below 22000 persons with individuals that has received complete vaccination rises above 21000 persons.

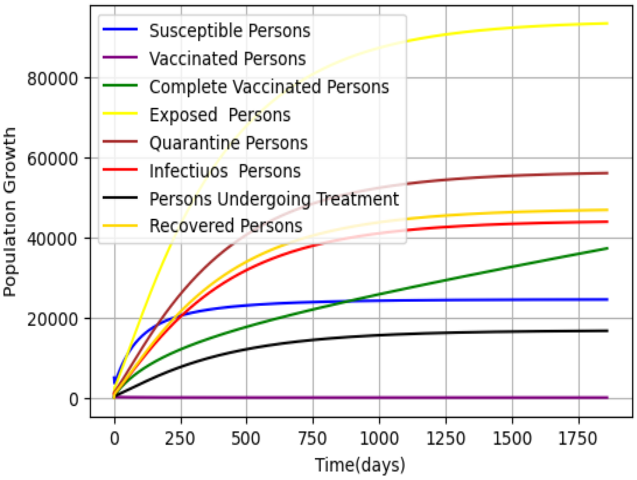

Figure 12: COVID - 19 model compartments with e1 = 0 and b2= 0.02 (5 Years without vaccine enlightenment)

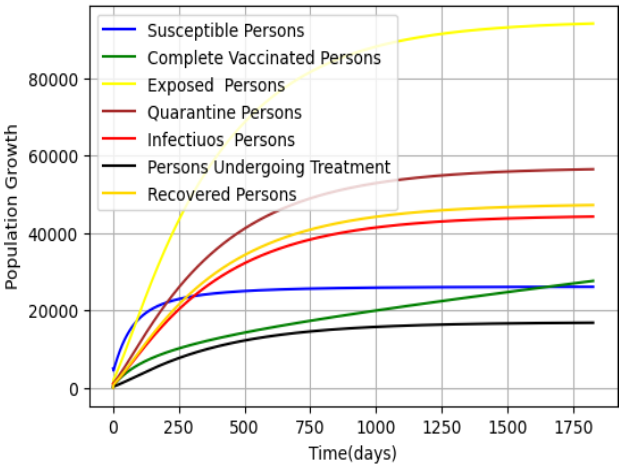

Figure 13: COVID - 19 model compartments with e1 = 0.5 and b2= 0.02 **(5** Years with vaccine enlightenment at 50%)

Comparing Figures 12 and 13, the gragh projected that the vaccinated population will rise to almost 40000 persons, susceptible population at 22000 in five years time with enlightenment for safe vaccination at 50%. While without enlightenment, the susceptible population will be 22000 persons with complete vaccination around 21000 persons.

Figure 14 and Figure 15 showed that complete vaccination population superseded exposed population and moved to almost 100,000 persons over the years at 50% enlightenment while it will be 60,000 at zero enlightenment level.

Inputting our model parameters as highlighted on Table 2 except change in possibility of vaccinated persons to be infected with COVID-19 at $b_1 = 0.0001$, the computed rate at which one infectious individual can generate infectious cases were given by $R_0 = 0.06055$. Results on model compartments are displayed on Figure 16 and Figure 17. This showed that at this rate of interaction, the infectious compartments and the subsequent compartments will become insignificant while COVID-19 will fizzle out in a short period of time. Also, comparing Figures 16 and 17, this result indicated that when rate of interaction with the virus is this low (one person per ten thousand) with the corresponding basic reproduction number as computed, enlightenment for vaccination become insignificant on the transmission of COVID-19. This result supports [15] theorem of reproduction number.

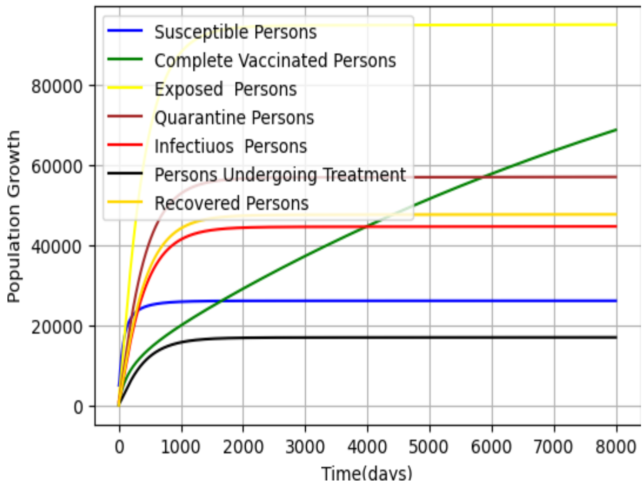

Figure 14: COVID - 19 model compartments with e1 = 0 and b2= 0.02 **(**22 Years without vaccine enlightenment)

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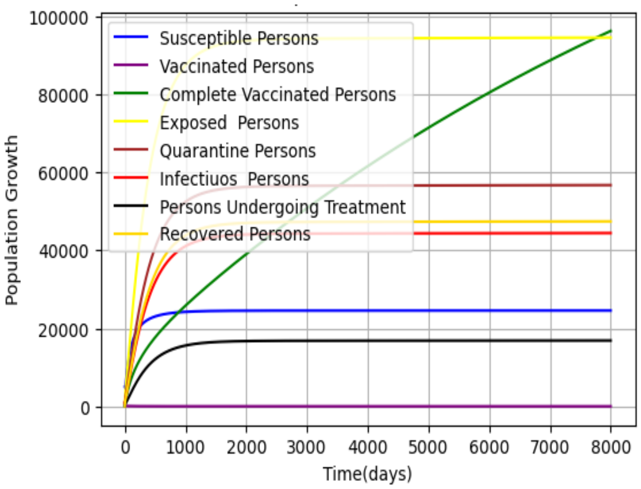

Figure 15: COVID - 19 model compartments with e1 = 0.5 and b2= 0.02 (22 Years with vaccine enlightenment at 50%)

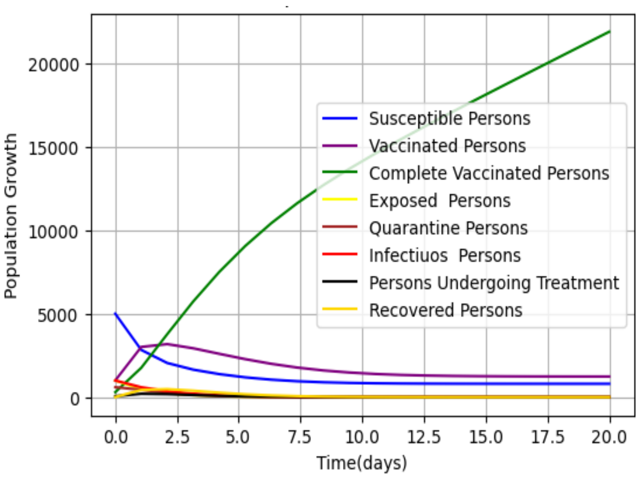

Figure 16: COVID - 19 model compartments with e1 = 0 and b2= 0.02 (Few days with contact at 0.0001, $e_1 = 0$)

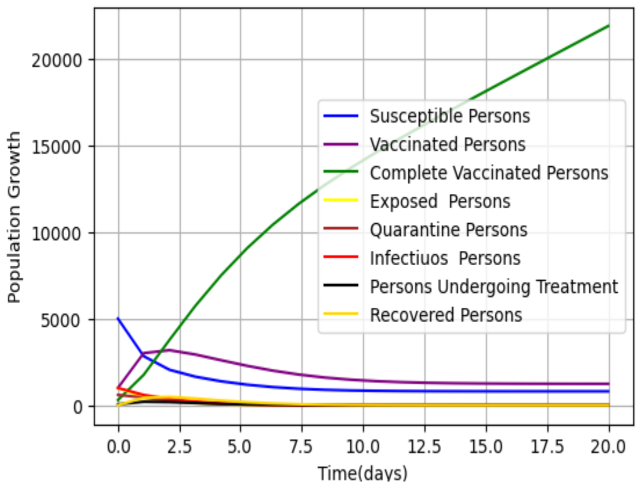

Figure 17: COVID - 19 model compartments with e1 = 0.5 and b2= 0.02 (Few days with Contact at 0.0001, $e_1 = 50\%$)

4.0 CONCLUSIONS

The study provided a model of the dynamics of COVID-19 transmission with control through enlightenment for the use of safe vaccines and treatment of infected persons. The study developed an eight compartment mathematical model of coronavirus disease 2019 transmission dynamics in the form of a system of differential equations. Considering vaccination, quarantine, isolation with treatment and natural recovery from the virus. These measures were advocated to control the spread of the virus and reduce human casualties. The model's preliminary analyses showed the positivity of the solution in the defined manifold. The DFE and EEP were obtained, with conditions for their stability established and discussed using NCDC statistics. The replication number of the disease was computed with the available statistics, considering WHO and NCDC advocacy for vaccines and drugs in use globally. The numerical analysis of the R_0 using enlightenment for the use of safe vaccines and treatment in isolation showed conditions to eradicate the disease. Also, to ensure optimal control of the disease spread, sensitivity analysis to obtain the best intervention strategies indicated the possibility for total eradication and prevention of an upsurge of the disease within the population using restriction of movement, full vaccination and public enlightenment for vaccination as major intervention strategies.

In summary, the results showed strong control of the disease through enlightenment for vaccination, isolation and use of effective safe vaccines. The result also portrayed that prevention of the spread through effective vaccination could be more effective than enlightenment for isolation without treatment.

REFERENCES

[1] World Health Organization, "How is it transmitted?," 2021. [Online]. Available: https://www.who.int/emergencies/diseases/novel-coronavirus-2019/question-and-answers-hub/q-a-detail/coronavirus-disease-covid-19-how-is-it-transmitted.

[2] World Health Organisation, "Coronavirus," Health Topics, 2020a. [Online]. Available: (2020). *Coronavirus*. Retrieved from https://www.who.int/health/topics/coronavir

https://doi.org/10.4314/njt.2025.3692

us. [Accessed 2022].

[3] World Health Organisation, "Situation Report – 90," 19 2020. [Online]. Available: https://www.who.int/docs/default-source/coronavirus-2019/situation-reports/20200419-sitrep-90-covid-1.

[4] World Health Organisation, "Coronavirus disease (COVID-19): Vaccines and vaccine safety," World Health Organization, 28 June 2023b. [Online]. Available: https://www.who.int/emergencies/diseases/novel-coronavirus-2019/question-and-answers-hub/q-a-detail/coronavirus-disease-(covid-19)-vaccines. [Accessed 2023].

[5] Nigerian Centre for Disease Control, "An update of COVID-19 outbreak in Nigeria," 26 February 2023. [Online]. Available: https://ncdc.gov.ng/themes/common/files/sitreps/4e947f95f2b5cd7826f0f42f44521439.pdf.

[6] World Health Organization, "WHO Coronavirus (COVID-19) Dashboard," 2023. [Online]. Available: https://covid19.who.int/.

[7] Iboi, E. A., Ngonghala C. N. and Gumel, A. B. "Mathematical modeling and analysis of COVID-19 pandemic in Nigeria.," *Mathematical bioscience and engineering*, 17(6), pp. 7192-7220, 2020.

[8] Abioye, A. I., Peter, O. J., Ogunseye, H. A., Oguntolu, F. A., Oshinubi, K., Ibrahim, A. A. and Khan, I. "Mathematical model of COVID-19 in Nigeria with optimal control," *Results in Physics*, 28(2021), 2021. https://doi.org/10.1016/j.rinp.2021.104598

[9] Okoye, C., Okofu, M. B., Benson, I. D., Nubila, N. K., Ejikeme, C. L., Aguegboh, S. N., Agbebaku, D. F. and Mbah, G. C. E. "A mathematical approach to understanding the effectiveness of control measures in mitigating coronavirus disease 19 (COVID-19) spread," *Academic Journal of Statistic and Mathematics*, 7(10), pp. 14–22, 2021.

[10] Bolaji, B., Omede, B. I., Odionyenma, U. B., Ojih, P. B. and Ibrahim, A. A. "Modelling the transmission dynamics of Omicron variant of COVID-19 in densely populated city of Lagos in Nigeria," *Journal of the Nigerian Society of Physical Sciences*, 5(2), pp. 1-13, 2023 https://doi.org/10.46481/jnsps.2023.1055.

[11] Duru, E. C., Mbah, G. C. E. and Alwell, U. "Numerical Simulations and Solutions of a Mathematical Model for Zika Virus Disease," *Applications of Modelling and Simulations*, 9, pp. 139-153, 2025.

[12] Mbah, G. C. E., Onah, I. S., Ahman, Q. O., Collins, O. C., Asogwa, C. C. and Okoye, C. "Mathematical modelling approach of the study of ebola virus disease transmission," *African Journal of Infectious Diseases (AJID)*, 17(1), pp. 10-26, 2022. https://doi.org/10.21010/Ajidv17i1.2

[13] Omame, A., Okuonghae, D., Nwafor, U. E. and Odionyenma, B. U. "A Co-infection model for HPV and Syphilis with Optimal Control and Cost-Effectiveness Analysis," *MedRxiv*, pp. 1-29, 2020. https://doi.org/10.1101/2020.09.20191635

[14] Moghadas, S. M., Gumel, A. B., McLeod, R. G. and Gordon, R. "Could condoms stop the AIDS epidemic?," *Journal of Theoretical Medicine*, 5(3-4), pp. 171-181, 2003. https://doi.org/10.1080/10273660412331315147

[15] Van den Driessche, P. and Watmough, J. "Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission," *Mathematical Biosciences*, 180(1-2), pp. 29-48., 2002.

[16] Atokolo, W. and Mbah, G. C. E. "Modeling the Control of Zika Virus Vector Population Using the Sterile Insect Technology," *Journal of Applied Mathematics*, 2020, pp. 1-12, 2020 https://doi.org/10.1155/2020/6350134.

[17] Fleming, W. H. and Rishel, R. W. "Deterministic and stochastic optimal control," *Bulletin of the American Mathematical Society*, 82, pp. 869-870, 1976.

https://doi.org/10.4314/njt.2025.3692