MODELS FOR PREDICTION OF STRUCTURAL PROPERTIES OF PALMNUIT FIBRE-REINFORCED CEMENT MORTAR COMPOSITES

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ABSTRACT
An analytical study was carried out to investigate the structural properties of palmnut fibre reinforced cement-based composites. Explicit expressions were derived for the flexural, compressive and elongation behavior of composites using a two-phase constitutive model and verified using results obtained from literature. The predicated and measured values were found to agree favourably. Both the predicted and measured composite specimens showed increasing flexural strength up to the optimum fibre volume fraction while the compressive strength decreases with increase in fibre volume fraction. The correlation coefficients were high showing the effectiveness of the two-phase constitutive models in the analysis and design of palmnut fibre-reinforced cement composites.

1.0 INTRODUCTION
Apart from food, the most basic problem of man is shelter. This problem is acute in Nigeria and many other developing countries due to inflation, unemployment, high growth rate of population and poor economic resources.

Corrugated aluminum, asbestos cement and galvanized iron roofing sheets although used in roofing, are very expensive and may constitute about 40% of the total cost of the building project and as a result cannot be afforded by the middle and low income earners. Apart from the high cost, there are certain shortcomings associated with the use of some of these materials. For example, asbestos dust from asbestos cement sheets results in health hazards such as asbestosis and mesothelioma.

In a bid to finding acceptable solution to the increasing inadequacy of housing delivery in developing countries, various local vegetable fibre-reinforced cement mortar composites have been explored as alternative roofing materials [1-12]. It is concluded from the findings that vegetable fibre-reinforced mortar is indeed a promising roofing material. However, the failure which occurs in vegetable fibre-reinforced mortar roofing projects are traceable to lack of or improper know-how transfer, to missing knowledge concerning material properties, production techniques and installation methods [12]. As a contribution towards filling this gap, empirical models were developed to predict some of the properties of palmnut fibre-reinforced cement composites. It is believed that research into the prediction of structural properties of vegetable fibre-reinforced cement composites will enhance optimum utilization of these materials.
2.0 DERIVATION OF GOVERNING EQUATIONS

Basic Assumption (Pakotiprapha et al [13])

1. The matrix and fibres are firmly bonded together.
2. The tensile stress is carried by the fibres alone.
3. The compressive stress is carried by the matrix alone.
4. The strain in composite equals strain in fibres and matrix.
5. The effect of Poisson ratio is neglected.
6. All fibres have an equal probability of being oriented at any direction to the load axis.
7. In bending, the elastic modulus in tension and compression are equal at uncracked range.

The total volume fraction of the materials equals unity.

\[ \lambda_m + \lambda_f = 1 \] (Pakotiprapha et al 1974 [13])

Where \( \lambda_m \) and \( \lambda_f \) are volume fractions of matrix and fibres respectively.

Consider a \( k^{th} \) fibre embedded in a matrix continuum as shown in Figure 1. Let the uniaxial tension, \( \sigma_c \) be applied perpendicular to the plane of the elemental material. The element strain is as shown in Figure 2. Let \( \theta_k \) be the angle of orientation of the \( k^{th} \) fibre with respect to the load axis.

From strain transformation,

\[ \varepsilon_f = \varepsilon_c \cos^2 \theta_k \] ...2.2

The cracking stress in the composite per unit area is given by

\[ \sigma_c = \sigma_m A_m + \sigma_f A_f \] (Constitutive Model – Pakotiprapha et al, 1974 [13]) ...2.3

In terms of the relative proportion of the individual sub-materials in the binary mixture, equation (2.3) is expressed as

\[ \sigma_c = \sigma_m \lambda_m + \sigma_f \lambda_f \] ...2.4

where \( A_m, A_f \) = Cross-sectional areas of matrix and fibres respectively.

\( \sigma_c \) = Cracking stress of composite.

\( \varepsilon_f, \varepsilon_c \) = Strain in fibre and composite respectively.

Parallel to stress axis, the component of stress of \( k^{th} \) fibre is \( \sigma_c \cos \theta_k \).

From equation (2.4),

\[ \sigma_c = \sigma_m A_m + \sum_{k=1}^{n} \sigma_f A_f \cos \theta_k, \quad n = 2 \] (Binary Mixture) ...2.5

Multiplying equation (2.5) by a unit length in the direction of stress gives

\[ \sigma_c = \sigma_m \lambda_m + \sum_{k=1}^{n} \sigma_f A_f \cos^2 \theta_k \] ...2.6

Since the area of fibre appearing in the arbitrary plane section of the composite is a function of the angle, \( \theta_k \), and fibre inclusion in matrix results in stress concentration in

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Figure 1: A composite element under uniaxial stress

Figure 2: Strain Response of a Composite Element in Tension
matrix, which reduces matrix strength. A factor $K_s$ is introduced to cater for reduced matrix strength.

$$K_s = \frac{1 - \lambda_f}{1 - \frac{\sigma_{fu}}{\sigma_f}}, \quad \sigma_{ec} = \sigma_m$$

(Modified equation, Greszczuk [14])

From fibre inclusion strength reduction effect on matrix,

$$\Sigma \sigma_f = \sigma_{cm} + \sum_{k=1}^{n} \sigma_{f_k} \lambda_{f_k} \cos^2 \theta_k$$

Where $\sigma_{cm}$, $\sigma_f$ = Matrix compressive strength and fibre tensile strength respectively. The composite element is bi-layered, and from the idealized stress-strain curve of Figure 3, composite failure in axial compression begins where the extreme compressive stress reaches $\sigma_{ec}$, where $\alpha = 0.90$ (ACI 1971 [15]). Then, from bilinear consideration, the composite compressive strength, $\sigma_{cc}$ is:

$$\Rightarrow \sigma_{cc} = 0.90K_s \sigma_{cm}(1 - \lambda_f) + \sum_{k=1}^{n} \sigma_{f_k} \lambda_{f_k} \cos^2 \theta_k$$

Parallel to stress axis, $\theta = 0$

$$\Rightarrow \sigma_{cc} = 0.90K_s \sigma_{cm}(1 - \lambda_f) + \sigma_f \lambda_f$$

Figure 3: Idealized Composite Compressive Stress-Strain Curve (AC1, 1971 [15])

From assumption number 3,

$$\Rightarrow \sigma_{cc} = 0.90K_s \sigma_{cm}(1 - \lambda_f)$$

Composite Flexural Strength

The composite flexural strength is predicted based on stress-strain diagrams shown in Figure 4. Moment on tension face for a unit width, $b$ of section is given by

$$M = \frac{f_{sb}bh^2}{3}$$

so that

$$f_{sb} = \frac{3M}{bh^2}$$

Using similar triangle,

$$\frac{h_c}{h_t} = \frac{\varepsilon_{cb}}{\varepsilon_{sb}} = \beta$$

where $\beta$ = Neutral axis parameter

But $h = h_t + h_c$ and $\frac{h}{h_t} = 1 + \frac{h_c}{h_t}$

so that,

$$h_t = \frac{\varepsilon_{sb} \cdot h}{\varepsilon_{sb} + \varepsilon_{cb}}$$

$$\Rightarrow f_{sb} = \frac{3M}{bh2} \left(1 + \frac{\varepsilon_{cb}}{\varepsilon_{sb}}\right)^2$$

Fiber inclusion in matrix reduces composite compressive strength and can be expressed as:

$$\Rightarrow \sigma_{cc} = \sigma_{cm} - \sigma_f = \sigma_{cm} - \lambda_f \sigma_{cm} = \sigma_{cm}(1 - \lambda_f)$$

In the tensile zone stresses are carried by fibres alone thus, $\sigma_u = \sigma_f$, but $\varepsilon_c = \sigma/E_{ct}$, $\varepsilon_{cb} = \sigma_{cm}/E_{cm}$ and $\varepsilon_{sb} = \sigma_f/E_{ct}$. Substituting these relationships in equation 2.15 gives
\[ f_{cb} = \frac{3M}{bh^2} \left( 1 + \frac{\sigma_{cu}}{\sigma_f} \left( 1 - \lambda \right) \right)^2 \]

For centre-point loading under simple support condition which was adopted in the experimental determination of the flexural strength of the fibre-cement mortar composite:

\[ M_{\text{max}} = \frac{PL}{4} \text{ thus,} \]

\[ f_{cb} = \frac{3PL}{4bh^2} \left( 1 + \frac{\sigma_{cu}}{\sigma_f} \left( 1 - \lambda \right) \right)^2 \]

Where

\[ \epsilon_{cb}, \epsilon_{tb} = \text{Composite compressive and tensile strain in bending respectively.} \]

\[ E_{ct}, E_{cc} = \text{Modulus of elasticity of composite in tension and compression respectively.} \]

\[ f_{cb} = \text{Flexural strength of composite} \]

\[ P = \text{Load at failure} \]

Figure 4 Stress-Strain Distribution
3.0 RESULTS AND DISCUSSIONS
The predictive models derived were verified using some experimental data obtained from literature [1].

3.1 Compressive strength
Results for the experimental and predicted model for composite compressive strength as a function of fibre volume fraction at different water-cement ratio are shown in Figure 5. The curves agree closely with correlation coefficients of 0.908 and 0.901 and standard errors of estimate of 2.69 and 2.23 for water/cement ratio of 0.4 and 0.6 respectively. F-statistic test was also conducted and the result show that both the experimental and the predicted values have the same variance thus, indicating good reliability of the model.

From the predicted curve it is seen that the compressive strength decreases with increase in fibre volume fraction. This trend may be attributed to the fact that the fibres in themselves can not resist axial compressive load and as such do not contribute to the compressive strength of the composite. Rather, under compressive type of loading the fibres may be viewed as a “filler” in the mortar matrix thus, introducing voids and discontinuity in the matrix with consequent reduction of strength.

The deviation of the experimental results from the predicted results may be attributed to some imperfections inherent in the simplifying assumptions made in the derivations.
Figure 5: Relationship Between Compressive Strength and Volume Fraction of Fibre.
3.2 Flexural Strength

Typical results for the experimental and predicted model for the flexural strength of the fibre-mortar composites are presented in Figure 6. The results agree closely with correlation coefficients of 0.976 and 0.90 and standard errors of estimate of 1.21 and 1.28 for water/cement ratio of 0.4 and 0.6 respectively. In addition, F-statistic test was conducted and the result show that both the experimental and the predicted values have the same variance thus, indicating good reliability of the model.

Both the experimental and predicted curves show that fibre inclusion in the matrix improves the flexural strength of the composite. The improvement is attributed to the ability of the fibres to arrest the crack growth and hence offering the composite a post-crack load carrying capacity [2]. In addition, an optimum volume fraction of fibre of 2% was observed in both the experimental and predicted model.

The predicted curves are a little lower than the experimental curves probably because the post-crack strength of the composite was ignored in the model derivations.

CONCLUSIONS

The following main conclusions are drawn from the study:-

(i) The two-phase constitutive model gave a very high correlation coefficient indicating the effectiveness of the model in the prediction of the flexural and compressive strength of palmnut fibre-reinforced cement mortar composites.

(ii) The optimum volume fraction of fibre is 2.0%.
REFERENCES