MECHANISM OF TORQUE PRODUCTION IN A COUPLED POLYPHASE RELUCTANCE MACHINE

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ABSTRACT

This paper studies the nature and mechanism of torque production in a coupled polyphase reluctance machine with two identical stator windings. For a graphical illustration of the torque mechanism, the permeance function approach is adopted in preference to the d–q axis analysis. The resultant mmf distributions on both halves of the machine are derived for asynchronous operation and found to be exerting an electromagnetic torque on their leading rotor pole axes in the anti-clockwise direction for the motoring mode and on the trailing pole axes for generator operation. It is shown that the machine develops synchronous torque while operating in the asynchronous mode and the sharing of the load torque between the coupled machine units is examined.

Key words: mmf, permeance, transposed auxiliary winding, flux distribution, flux density.

List of Principal Symbols

\( m_0 \) – Instantaneous value of the magnetizing mmf

\( M_0 \) – Peak amplitude of the instantaneous magnetizing mmf

\( X \) – Angular distance measured from the references axis, which is taken as the center line of the stator poles.

\( P_A \) – Rotor permeance distribution for unit area of air-gap of machine A half

\( P_B \) – Rotor permeance distribution for unit area of air-gap of machine B half

\( P_0 \) – Constant component of the rotor permeance distribution

\( P_V \) – Variable component of the rotor permeance distribution

\( \omega_0 \) – Supply angular frequency

\( \omega \) – Rotor speed

\( t \) – Time in seconds

\( V \) – Applied voltage to the primary winding

\( e_2, E_2 \) – Instantaneous voltage induced in the auxiliary winding and its rms value

\( E_{2m} \) – Peak amplitude of the instantaneous voltage \( e_2 \)

\( B_{A0} \) – Flux density produced in section A half of the machine with auxiliary winding on
Open circuit

\( B_{B0} \) – Flux density produced in section B half of the machine with the auxiliary winding on open circuit.

\( i_2, I_2 \) – Instantaneous current circulating in the auxiliary winding and its rms value

\( \phi \) - Impedance angle

\( m_1, M_1 \) – Instantaneous mmf distribution of the primary winding and its peak amplitude

\( m_2, M_2 \) - Instantaneous mmf distribution in the auxiliary winding and its peak amplitude

\( B_{2A} \) - Flux density distribution in section A half of the machine with auxiliary winding closed

\( B_{2B} \) - Flux density distribution in section B half of the machine with auxiliary winding closed

\( B_{A0} - B_{A0} \) – Average flux density linking the Auxiliary winding with auxiliary winding on open circuit.

\( B_{2T} \) – Total flux density linking the auxiliary winding when it is closed

\( S_a \) - Slip of the auxiliary winding field with respect to the rotor

\( S_p \) - Slip of the primary winding field with respect to rotor

\( M_{RA} \) – Resultant mmf on the rotor pole tip of section A half of the machine

\( M_{RB} \) – Resultant mmf on the rotor pole tip of section B half of the machine

\( M' \) – Vector sum of \( M_0 \) and \( M_1 \)

\( \sigma \) - Overall primary input power factor angle.

1.0 INTRODUCTION

Conventional polyphase reluctance machines operate at a fixed speed that is directly and simply related to the supply frequency [1,2]. With some modifications in its configuration, a polyphase reluctance machine can operate asynchronously [3,4,5]. In one arrangement, for an idealized machine, there are two sets of stator windings, which are both skewed, one, skewed 180 electrical degrees and the other skewed −180 electrical degrees [6]. One set of windings, the main (primary) is connected to the source, whilst the other set, that is the auxiliary (secondary) winding is short-circuited. The roles of the two windings can be interchanged. An alternative configuration is to have the primary not skewed, whilst the rotor is skewed 180 electrical degrees and the auxiliary winding skewed 360 electrical degrees in the same sense. In a practical, realistic machine, on account of the conductor being located in slots, the skew can only be accomplished in discrete steps requiring therefore that the machine be assembled in two or more magnetically isolated stacks [3,6].

In a two-stack machine with un-skewed primary windings, the d-axes of the rotors of the two halves of the machine are mutually displaced electrically by 90° and the axes of the two halves of the auxiliary winding coils displaced
electrically by 180° [3,7]. The machine so described is known as transfer field machine [3]. The transfer field machine unlike the Broadway’s configuration [4] has two electrically isolated stator (main and auxiliary) windings, which carry the excitation current and circulating current respectively. The (2s-1)ω₀ low frequency current is confined in the auxiliary winding without interfering with the supply. This paper set forth to study the nature and mechanism of torque production in coupled polyphase reluctance machines with two identical stator windings popularly known as transfer field (TF) machines. The cyclic variation of the mutual coupling between the primary winding and auxiliary winding coils is the basis of operation of the machine.

The self-inductances of each phase winding is unaffected by the rotor position. In fact, these relationships between the primary and auxiliary windings are similar to the relationship between the primary and secondary windings of an induction machine or between the phase winding and excitation winding of a round rotor synchronous machine. A TF machine was constructed and successfully tested by one of the authors [3]

2.0 MACHINE MODEL

Fig. 1 shows the per phase schematic diagram of the TF machine. Each machine half is similar in features to the conventional synchronous machine. The major unorthodox features of the machine are:

a) the stator and rotor are arranged in two identical halves; and hence the machine may be treated as two separate reluctance machines whose stator windings are connected in series.
b) There are no conductors in the rotor, including damper windings.
c) The pole axes of the two pole halves are mutually in space quadrature.

d) There is second set of polyphase stator windings (auxiliary winding) whose conductor side are shifted electrically by 180° (transposed) in passing through one section of the machine to another. The main and auxiliary windings are identical in every respect and occupy the same electrical positions in the stator slots thus ensuring a perfect coupling between the windings.

3.0 PRINCIPLE OF OPERATION AND ANALYSIS

The main (primary) winding of the machine is connected to the supply voltage V and draws an excitation current I₀ at the source frequency ω₀ when the auxiliary winding is on open circuit, which produces an mmf whose distributions on both halves of the machine A, B can be expressed as

\[ m_o = m_o \cos (x - \omega_o t) \]  

(1)

The air-gap permeance distribution as seen by the rotor in the first half A, of the machine, may be expressed as:

\[ P_A = P_0 + P_V \cos 2(x - \omega t) \]  

(2)

where ω is the speed of the rotor.
The air-gap permeance distribution in the second half B, whose auxiliary winding is 180° displaced from that of A (transposed) will be given by

\[ P_B = P_o + P_V \cos 2(x - \omega t - 90^\circ) = P_o - P_V \cos 2(x - \omega t) \]  

(3)

\[ M_0P_0 \cos(x - \omega_0 t) + \frac{M_BP_V}{2} \]

*\( \cos(x + (\omega_0 - 2\omega) t) \) + third space harmonics 

(5)

Similarly, the corresponding flux density distribution produced in section B part of the machine is:

\[ B_{B0} = M_0P_0 \cos(x - \omega_0 t) - \frac{M_BP_V}{2} \]

\[ \cos(x + (\omega_0 - 2\omega) t) + \text{third space harmonics} \]  

(6)

The average flux density as seen by the main winding is equal to the combination of equations 5 and 6 and given by

\[ B_{ap} = M_0P_0 \cos(x - \omega_0 t) \]  

(7)

with a rotation in the positive anti-clockwise sense.

The direction of emf \( E_1 \) induced in the main winding conductor in both halves of the machine A, B such as “a1” and “a1’” by this flux is as shown in fig 2a in full lines. These emfs \( E_1 \) are both equal in magnitude and in time phase in both sections of the machine and will oppose the supply voltage \( V \) in both sections.

The flux density produced by this mmf at the instant when its axis coincides with the pole axis of the rotor of section A part of the machine is given by [1 - 7]:

\[ B_{AO} = m_o P_A \]  

(4)

The emfs induced by the third space harmonics cancel out if the windings are star-connected. If, however, the windings are desired to be connected in delta, the winding pitch must be chosen such that the
emf induced by harmonics of flux is eliminated.

The average value of the second components of the flux in equations 5 and 6 (i.e. $\frac{M_0 P_v}{2} \cos(x + (\omega_0 - 2\omega)t)$ is zero.

However, the average of this flux as seen by the other set of windings (auxiliary) whose conductors are transposed between the two machine halves is given by:

$$B_{A0} - B_{B0} = \frac{M_0 P_v}{2} \cos(x + (\omega_0 - 2\omega)t)$$

This flux density distribution rotates in the negative clockwise direction for $\omega < \omega_0/2$.

The direction of emf $E_2$ induced in the auxiliary windings by this flux will be in anti-phase in both halves of the machine because of the transposition of the auxiliary windings as shown in the dotted circles in fig. 2b. It therefore follows that in section A half, $E_2$ will be diametrically opposed to $E_1$ and in section B half $E_2$ will be in phase with $E_1$.

4.0 SHORT CIRCUITING OF THE AUXILIARY WINDINGS

If the auxiliary windings are short-circuited, emf will be induced in it at $(\omega_0 - 2\omega)$ frequency and is given by:

$$e_2 = E_{2m} \cos(\omega_0 t - \pi/2) \text{------ (10)}$$

This induced emf $e_2$ will circulate current $i_2$ say at the same frequency in the short-circuited winding and this induced current is inversely proportional to its leakage impedance and may be expressed as:

$$i_2 = \frac{E_{2m} \cos((\alpha_0 - 2\omega)t - \pi/2)}{\sqrt{r_a^2 + ((\alpha_0 - 2\omega)L)^2}}$$

$$= I_{2m} \left(\cos(\omega_0 t - \pi/2) - \phi\right) \text{------ (11)}$$

$r_a$ is the resistance of the winding, $L$ is the inductance of the winding and $\phi$ is the impedance angle.

The distribution of mmf in the auxiliary winding of one machine half (section A say) due to this current has the form:

$$m_2 = M_2 \cos(x + (\omega_0 - 2\omega)t - \pi/2 - \phi) \text{------ (9)}$$

In section B, the corresponding auxiliary winding mmf will be (due to transposition of auxiliary windings):

$$m_2 = -M_2 \cos(x + (\omega_0 - 2\omega)t - \pi/2 - \phi) \text{------ (13)}$$

The flux density distribution in section A half due to its rotating mmf will be given by

$$B_{2A} = m_2(P_0 + P_v \cos2(x - \omega_0 t))$$

$$= M_2 \cos(x + (\omega_0 - 2\omega)t - \pi/2 - \phi)(P_0 + P_v \cos2(x - \omega_0 t)) \text{------ (14)}$$

Similarly, in section B, the corresponding flux density distribution is

$$B_{2B} = -m_2(P_0 - P_v \cos2(x - \omega_0 t))$$

$$= -M_2 \cos(x + (\omega_0 - 2\omega)t - \pi/2 - \phi)(P_0 - P_v \cos2(x - \omega_0 t)) \text{------ (15)}$$

Therefore, the total flux density over the two sections of the machine is the combination of equations 14 and 15 and given by

$$B_{2T} = M_2 \cos(x + (\omega_0 - 2\omega)t - \pi/2 - \phi)(P_0 + P_v \cos2(x - \omega_0 t)) - M_2 \cos(x + (\omega_0 - 2\omega)t - \pi/2 - \phi)(P_0 - P_v \cos2(x - \omega_0 t)) \text{------ (16)}$$

$$= M_2P_v \cos(x - \omega_0 t + \pi/2 + \phi) + \text{third space harmonics} \text{------ (17)}$$
The third harmonics would have no effect if the winding is star connected. The presence of $I_2$ in the auxiliary winding leads to an additional current $I_1$ to be drawn by the primary in a manner akin to what obtains in a transformer or an induction motor in order to neutralize the effect of the auxiliary current $I_2$. The primary current $I_1$ produces a primary flux needed to neutralize the secondary flux.

5.0 MMF BALANCE

The auxiliary winding load current mmf is $m_2 = M_2 \cos(x + (\omega_0 - 2\omega) t - \pi/2 - \phi)$ --- (18)

The flux of $m_2$ linking the primary winding may be expressed as:

$$B_{2T} = m_2 P_v \cos 2(x - \omega t)$$

$$= M_2 P_v \cos(x + (\omega_0 - 2\omega) t - \pi/2 - \phi) \cos 2(x - \omega t)$$

$$= M_2 P_v \cos(x - \omega_0 t + \pi/2 + \phi) + \text{third space harmonics}$$

----- (19)

This flux must be opposed and balanced by the primary load current flux. The primary mmf may be expressed as $m_1 = M_1 \cos(x - \omega_0 t - \Psi)$ ----- (20)

The self-flux of $m_1$ is given by $m_1 P_0 = M_1 P_0 \cos(x - \omega_0 t - \Psi)$ ----- (21)

For mmf balance

$$M_1 P_0 \cos(x - \omega_0 t - \Psi) = -M_2 P_v \cos(x - \omega_0 t + \pi/2 + \phi)$$

$$= M_2 P_v \cos(x - \omega_0 t - \pi/2 + \phi)$$

----- (22)

Thus, $M_1 P_0 = M_2 P_v$ ----- (23)

If $\Psi = \pi/2 - \phi$

Therefore

$$m_1 = M_1 \cos(x - \omega_0 t - \pi/2 + \phi)$$

(24)

The roles of primary and auxiliary windings can be exchanged. If the auxiliary winding is used as the primary and if the mmf is $M_2$, then the primary, now the secondary will have mmf $M_1$.

Therefore $M_1 P_v = M_2 P_0$ ----- (25)

Combining this with $M_1 P_0 = M_2 P_v$ ----- (26) gives

$$|M_1| = |M_2|$$

----- (27)

Fig. 3 at a certain instant in time shows the mmf distribution phasors in the two halves of the TF machine.

![Fig. 3a: Section A part of the machine showing mmfs. $m_0$, $m_1$, and $m_2$](image)

![Fig. 3b: Section B Part of the machine showing mmfs. $m_0$, $m_1$, and $m_2$](image)
In section A, \( M_1 \) and \( M_2 \) are equal in magnitude but in anti-phase whilst in section B \( M_1 \) and \( M_2 \) are equal in magnitude and in phase because of the transposition of the machine windings in section B.

### 5.1 PER UNIT SLIP

The slip of the primary winding field with respect to the rotor is given by

\[
S_p = \omega_0 - \omega = \omega_0 - (1 - s) \omega_0 = s \omega_0 \quad (28)
\]

Similarly, the corresponding auxiliary winding field slip with respect to the rotor is given by

\[
S_a = (2 \omega - \omega_0) - \omega = \omega_0 - \omega = -s \omega_0 \quad (29)
\]

Equations 28 and 29 imply that the primary winding mmf \( M_1 \) and the excitation current mmf \( M_0 \) will rotate at the speed \( s \omega_0 \) with respect to the rotor whilst the auxiliary winding mmf \( M_2 \) will rotate at the speed \( -s \omega_0 \) with respect to the rotor. Thus the combination of \( M_0 \) and \( M_1 \) (\( M_0 + M_1 = M' \)) rotate at the same speed as \( M_2 \) relative to the rotor but in the opposite direction.

### 6.0 TORQUE MECHANISM

When the auxiliary winding is on open circuit, the machine does not develop torque because:

i) the absence of a rotor winding which would produce induction motor type torque and

ii) the impedance of the winding does not change with rotor angular position, so there is no reluctance torque.

However, the component of the primary winding flux that induces emf in the auxiliary winding rotates in the negative sense relative to the rotating magnetizing mmf of the primary winding. That is, \( M_0 \) and \( M_1 \) rotate at the speed \( s \omega_0 \) while the auxiliary current mmf \( M_2 \) rotates at the same speed relative to the rotor but in the opposite sense. Consider an instant in time when the axes of the magnetizing mmf \( M_0 \) coincides with the d-axis of the rotor-pole in machine A and hence with q-axis of the machine B, both being in space quadrature. The magnetizing flux in machine A will be maximum, and that in machine B negligible compared to that of machine A. The effective magnetizing flux would therefore be almost entirely of that produced by machine A. In fig. 4a, shows section A part of the machine. The mmf phasors \( M_1 \) and \( M_2 \) are shown to be equal in magnitude but in phase opposition and with the resultant of \( M_0 \) and \( M_1 \) producing \( M' \) (i.e. \( M_0 + M_1 = M' \)), which rotates at the speed \( s \omega_0 \), whilst \( M_2 \) rotates at \( -s \omega_0 \). The resultant mmf in section A part of the machine \( (M_{RA} = M' + M_2) \) acts at the leading pole tip of the rotor as shown in fig. 4a. In section B half of the machine, the direction of \( M_2 \) phasor would have reversed due to the transposition of the auxiliary winding.

Hence, the phasors \( M_1 \) and \( M_2 \) will now be in phase and the resultant mmf \( (M_{RB} = M' + M_2) \) acts at the leading rotor pole tip. Since in both halves of the T – F machine A,
B respectively, their resultant mmfs, $M_{RA}$ and $M_{RB}$ act at the leading pole tips, consequently, their tangential components will exert a reluctance torque in the positive anti-clockwise sense in both halves of the TF machine which will pull the rotors forward.

![Fig. 4a: Section A part of the machine showing the resultant mmf $M_{RA}$ acting on the leading rotor pole tip](image)

It is obvious that the intensity of the resultant mmfs and hence flux is greater in machine B than in machine A at this instant $t_0$ say as depicted in fig. 4a and 4b. Thus, machine B produces the greater proportion of the net torque of the TF machine. As time progresses, $M_2$ and $M_1$ will be moving apart in machine B, thus reducing (weakening) the resultant mmf $M_{RB}$; but moving closer in machine A, thereby increasing (strengthening) the resultant mmf $M_{RA}$ and hence flux at this instant $t_2$ say. The combination of $M_1$ and $M_2$ will coincide at a point leading the axis of the rotor pole by $\phi$ after each of them has traveled a distance of $\pi/2$ electrical radians. Since torque is directly proportional to the square of mmf and hence flux, each section of the machine that produces the greater proportion of the flux will produce the greater proportion of the net torque that transfers cyclically from one machine half to the other. When the torque in one machine half is increasing the torque from the other half is decreasing while the net torque remains constant. In fact when the torque in one half is maximum, it will be zero in the other. This phenomenon can be explained quantitatively by considering the combination mmfs in the air gaps of each machine half of the TF machine. In section A machine half, the combination mmfs is given by

$$M_{mfs_A} = M_0 \cos(x - \omega_0 t) + M_1 \cos(x - \omega_0 t - \pi/2 + \phi) + M_2 \cos(x + (\omega_0 - 2\omega) \pi/2 - \phi)$$

$$(30)$$

$$= M' \cos(x - \omega_0 t - \sigma) + M_2 \cos(x + (\omega_0 - 2\omega) t - \pi/2 - \phi)$$

$$(31)$$

$$= (M' - M_2) \cos(x - \omega_0 t - \sigma) + M_2 \cos(x - \omega_0 t \sigma) + M_2 \cos(x + (\omega_0 - 2\omega) - \pi/2 - \phi)$$

$$(32)$$

$$= (M' - M_2) \cos(x - \omega_0 t - \sigma) + 2M_2 \cos(x - \omega t - 1/2 (\sigma + \phi + \pi/2)) * \cos ((\omega - \omega_0) t - 1/2 (\sigma - \phi - \pi/2))$$

$$(33)$$
where $\sigma$ is the primary input power factor angle.

The torque component of equation 33 is a wave that rotates around the air gap at the speed of the rotor $\omega$, whose amplitude is modulated at slip frequency by $\cos((\omega - \omega_0)t - \frac{1}{2}(\sigma - \phi - \pi/2))$ and can consequently impart a reluctance torque on the rotor in the forward anticlockwise direction.

The first component of equation 33 cannot produce torque since it rotates at a speed different from that of the rotor.

Similarly, the mmf combination in section B half of the machine is given by

$$Mmf_{SB} = (M' - M_2) \cos(x - \omega_0 t - \sigma) + 2M_2 * \sin(x - \omega_0 t - \frac{1}{2}(\sigma + \phi + \pi/2)) * \sin((\omega - \omega_0) t - \frac{1}{2}(\sigma - \phi - \pi/2))$$

Analogously, the second component of equation 34 is a wave that rotates around the air-gap at the speed of the rotor $\omega$ and can consequently impart a reluctance torque on the rotor.

It follows from equations 34 and 35 that the amplitude of each torque producing mmf in the two halves of the machine has the same space phase displacement with respect to the corresponding rotor axis and furthermore, the variation of the amplitude of the waves are in time quadrature. The physical meaning of this is that the torque swings cyclically between the two sections of the machine, while the net torque remains constant. The machine’s self-starting characteristics can be inferred from the gross mmfs in both halves of the machine, i.e. equations 33 and 34 which are non-zero at starting (i.e. at $\omega = 0$)

### 6.1 EFFECT OF SECONDARY (AUXILIARY) WINDING POWER FACTOR

Suppose the auxiliary winding is loaded such that the power factor is reversed as shown in fig. 5. It will be observed that the axes of the resultant mmfs $M_{RA}$ and $M_{RB}$ in both halves of the TF machine will now intersect at the trailing rotor poles tips as shown in fig. 5 and thus leading to an electromagnetic torque in the opposite direction to the rotation of the rotor. The machine in this case will require an external mechanical agency as a prime mover and this corresponds to generator operation mode.

![Fig. 5a: The mmf phasors of section A part of the machine illustrating generator operation](image)

![Fig. 5b: The mmf phasors of section B part of the machine illustrating generator operator](image)

### 6.2 TORQUE/SPEED CHARACTERISTIC
The magnitude of the emf induced in the auxiliary winding and hence the current $I_2$ in the winding is proportional to the velocity $(\omega_0 - 2\omega)$ at which the flux cuts the auxiliary conductors. The auxiliary current and hence the torque will therefore be zero when $\omega = |\omega_0|$. However, a torque can be developed at this speed if auxiliary windings are fed externally with dc to produce a stationary field. Thus the asynchronous torque/speed characteristic of the machine will resemble that of an induction motor with half as many stator magnetic poles as shown in fig. 6.

![Fig. 6: Torque Speed Curve of the TF machine](image)

Even though the torque speed characteristic resembles that of an induction motor, it should be noted that the action is due to reluctance effects and not due to alignment of primary and secondary fields as in the induction motor. It follows that when motoring, the axis of the torque-producing wave leads the axis of the rotor pole and lags behind it for the generating mode. In the speed range, $0 < \omega < \frac{1}{2}\omega_0$ the machine is motoring and in the range $\frac{1}{2}\omega_0 < \omega < \omega_0$ the machine is generating.

### 7.0 CONCLUDING REMARKS

The mechanism of torque production in a TF machine has been analytically and illustratively presented. Although the torque speed characteristic of a TF machine is similar to that of an induction machine with half as many stator magnetic poles, the characteristic is due to reluctance effects as distinct from alignment of fields as in induction machines.

This is to be expected, since all the windings of the TF machine are on the stator side and therefore alignment of fields cannot apply. This reluctance effect phenomenon can be used to explain the mechanism of torque production in single stack asynchronous reluctance machine having two stator windings with different pole numbers, which also differ from the rotor pole number [8]. Apart from the asynchronous operation of the device, synchronous operation at $\frac{\omega_0}{2}$ with dc excitation is also possible. With some modifications in the configuration, operation in the pure reluctance mode is possible.

Since the torque of the device is due to cyclic variation of the mutual coupling between the primary and auxiliary winding coils, individually therefore, none of the units of the coupled machines comprising the TF machine can develop a reluctance torque while operating alone. Torque can only be developed when they are mechanically coupled together.

The operation of the machine resembles that of two coupled single-phase reluctance motors acting in time quadrature.

The auxiliary windings provide a loop in which the $(2s - 1)\omega_0$ harmonics,
necessary for torque production in the asynchronous mode can circulate without interfering with the supply. The auxiliary windings also carry the dc currents required for synchronous operation at $\omega_0/2$.

REFERENCES


