COMPARATIVE STUDY OF VLASOV AND EULER INSTABILITIES OF AXIALLY COMPRESSED THIN-WALLED BOX COLUMNS

J. C. EZEH¹ AND N. N. OSADEBE²

1. Civil Engineering Department, Federal University of Technology, P.M.B. 1526, Owerri, Nigeria.
2. Civil Engineering Department, University of Nigeria, Nsukka.

ABSTRACT

Through experimental studies, Vlasov showed that Euler’s critical load formula cannot be directly applied to the buckling analysis of thin-walled closed columns. In this study, Vlasov’s displacement model with modification by Varbanov and Euler’s elastica model were used in a comparative study to determine the flexural buckling strength of single-cell doubly symmetric thin-walled box columns with different boundary conditions. The study involved a theoretical formulation based on Vlasov’s theory as modified by Varbanov and implemented the associated displacement model in analyzing flexural buckling modes. Euler’s critical load formula was used to solve the same set of problems and the results thereof were compared with those obtained from Vlasov’s model. The flexural behaviour showed that for all three sets of boundary conditions considered, the critical load due to flexure about the oz-axis will control design in both models. Comparison with the Euler critical load results showed that Euler’s model underestimated the critical buckling load by 67.53% for hinged-hinged, 67.11% for clamped-hinged and 66.11% for clamped-clamped boundary conditions respectively. For bending about the oy-axis, the underestimation ranged from 51.14% for hinged-hinged, 50.33% for clamped-hinged to 48.52% for clamped-clamped boundary conditions. The results show that for single-cell doubly symmetric box columns, the Euler buckling strength should be increased by about 100% to 200% to obtain the Vlasov buckling strength. The actual percentage depends on the axis of symmetry and the boundary conditions under consideration.

Key words: Euler’s Model, Flexural Buckling, Single-Cell Section, Thin-Walled Column, Vlasov’s Model.

NOTATIONS:

\( U_i(x) \): Longitudinal displacements function due to flexure about oy- and oz-axes and warping due to torsion about ox-axis.

\( V_i(x) \): Transverse displacements function due to flexure about oy- and oz-axes, torsion about ox-axis, and distortion of the cross-section.

\( \varphi_i(s) \): Generalized longitudinal strain fields due to flexure about oy- and oz-axes, and warping torsion about ox-axis.

\( \varphi_i'(s) \): First derivative of the longitudinal strain fields with respect to the profile coordinate, \( S \).

\( \psi_i(s) \): Generalized transverse strain fields due to flexure about oy- and
INTRODUCTION

Thin-walled structures consist of a wide and growing field of engineering application which seek efficiency in strength and cost by minimizing material [1]. Multitude of research efforts have recently been invested in thin-walled structures because of the complexity of their behaviour, their natural optimization characteristics and the need for brief but accurate and reliable design methods [2]. Thin-walled closed structures are very economical as structural elements due to their light weight, and their high flexural and torsional rigidity [3-6]. However, owing to the thinness of their walls, these structures appear to have low resistance against buckling, consequently their instability problems need some careful and in-depth study before their reliable design as compression elements can be accomplished [5]. Closed cross-section thin-walled steel columns have at least three competing instability modes: flexural, torsional and distortional buckling [6].

Backing his theory up with experimental studies, Vlasov showed that Euler’s critical load formula cannot be directly applied to the buckling analysis of thin-walled closed columns because the formula was derived on the basis of elementary beam theory which does not embrace cross section warping and distortion [7]. Subsequent research efforts have been quoting this finding by Vlasov, but none has studied the level of relationship between flexural buckling strengths obtained from the two models. Strict application of Vlasov’s equation for the analysis of thin-walled structures leads to large number of kinematic unknowns in form of displacement functions. As a result of this problem, Varbanov [4] showed that by using generalized strain fields, the number of kinematic unknowns was drastically reduced. Generalized strain fields are strain fields chosen in such a way that they constitute linear combinations of the unit (elementary) strain fields used by Vlasov [8].

The main motivation for this present study is the need to establish simple closed-form relationship between flexural buckling strengths obtained using Vlasov’s displacement model as modified by Varbanov and that obtained using Euler’s elastic model for single-cell thin-walled box columns. Availability of such simple relationship will enable an accurate estimate of Vlasov’s flexural buckling loads for single–cell box columns when a more direct Euler’s value is known.
FORMULATION OF THE EQUILIBRIUM EQUATION:

Figure 1 shows an axially compressed thin-walled hollow column, the generated stress resultants and the box cross-section parameters. Using Lagrange’s principle, Vlasov [8] expressed the displacements in the longitudinal and transverse directions, $u_{(x,s)}$ and $v_{(x,s)}$ of a thin-walled closed structure in series form as follows:

$$u_{(x,s)} = \sum_{i=1}^{m} U_i (x) \psi_i (s)$$

$$v_{(x,s)} = \sum_{k=1}^{n} V_k (x) \psi_k (s)$$

Vlasov’s formulation yields $(m + n)$ second order differential equations. Later work by Varbanov [4] showed that $m$ and $n$ can be limited to four by using generalized strain fields. Using equations (1) and (2) and basic stress-strain relations of the theory of elasticity, the expressions for normal and shear stresses become:

$$\sigma_{(x,s)} = E\varepsilon_x = E \sum_{i=1}^{m} U_i'(x) \psi_i (s)$$

$$\tau_{(x,s)} = G\gamma_{xs} = G \sum_{i=1}^{m} U_i (x) \psi_i'(s)$$

The bending moment induced by distortion is given by:

$$M_{(x,s)} = \sum_{k=1}^{n} M_k (s) \psi_k (s)$$

The potential energy of an axially loaded thin-walled closed structure is given by:

$$\pi_p = S - W$$

For the structure under consideration, the strain energy and work done by the external load are given by:

$$S = \frac{1}{2} \int_L \int_S \left[ \sigma_{(x,s)} \varepsilon_{(x,s)} + \tau_{(x,s)} \gamma_{(x,s)} \right] dx ds$$

$$W = \frac{1}{2} \int_L \int_S P \psi_{(x,s)} \frac{M^2}{EI} dx ds$$

Substituting equations (7) and (8) into equation (6) and simplifying, we obtained the
total potential energy functional as:
\[ \pi_p = \frac{1}{2} \int_L \left[ E \sum_{i=1}^{m} \sum_{j=1}^{m} a_{ij} U_i(x) U_j(x) + \right. \\
\left. G \sum_{i=1}^{m} \sum_{j=1}^{m} b_{ij} U_i(x) U_j(x) + \right. \\
\left. + G \sum_{i=1}^{m} \sum_{r=1}^{n} c_{ir} U_i(x) V_r'(x) + \right. \\
\left. + G \sum_{j=1}^{m} \sum_{k=1}^{n} c_{jk} U_j(x) V_k'(x) + \right. \\
\left. + G \sum_{k=1}^{n} \sum_{r=1}^{n} m_{kr} V_k'(x) V_r'(x) + \right. \\
\left. E \sum_{k=1}^{n} \sum_{r=1}^{n} s_{kr} V_k(x) V_r(x) - \right. \\
\left. E \sum_{k=1}^{n} \sum_{r=1}^{n} h_{kr} V_k'(x) V_r'(x) \right] dx \]
where
\[ a_{ij} = \int_S \varphi_i(s) \varphi_j(s) \kappa(s) ds \]
\[ b_{ij} = b_{ji} = \int_S \varphi_i'(s) \varphi_j'(s) \kappa(s) ds \]
\[ c_{ir} = c_{ri} = \int_S \varphi_i'(s) \psi_r(s) \kappa(s) ds \]
\[ c_{jk} = c_{kj} = \int_S \varphi_j'(s) \psi_k(s) \kappa(s) ds \]
\[ m_{kr} = m_{rk} = \int_S \psi_k(s) \psi_r(s) \kappa(s) ds \]
\[ h_{kr} = h_{rk} = \int_S \psi_k'(s) \psi_r'(s) ds \]
\[ s_{kr} = s_{rk} = \frac{1}{E} \int_S \frac{M_k(s)}{E} M_r(s) ds \]

The total potential energy functional \( \pi_p \) has stationary values if the following Euler-Lagrange differential equations are satisfied:
\[ \frac{\partial F}{\partial U_j} - \frac{d}{dx} \left( \frac{\partial F}{\partial U_j'} \right) = 0 \]

Where,
\[ \pi_p = F \left( U_i, U_j, V_k, V_r, U_i', U_j', V_k', V_r' \right) \]

Using equations (11) and (12) on equation (9) and noting that for the thin-walled closed column under consideration, \( m=3 \) and \( n=4 \), we obtained the governing equations of equilibrium as:
\[ \gamma \sum_{i=1}^{3} \sum_{j=1}^{3} a_{ij} U_i''(x) - \sum_{d=1}^{3} \sum_{j=1}^{3} b_{ij} U_d(x) - \sum_{r=1}^{4} \sum_{r=1}^{4} c_{jr} V_r'(x) = 0 \]
\[ \sum_{d=1}^{3} \sum_{r=1}^{4} c_{dr} U_d'(x) + \sum_{k=1}^{4} \sum_{r=1}^{4} m_{kr} V_k'(x) - \gamma \sum_{k=1}^{4} \sum_{r=1}^{4} s_{kr} V_k(x) = 0 \]

Where, \( k_{kr} = \left( m_{kr} - \frac{p}{G} h_{kr} \right) \)

GENERATION OF THE STRAIN MODES AND DETERMINATION OF COEFFICIENTS OF THE GOVERNING EQUATIONS OF EQUILIBRIUM:
Considering the nature of loading (axial compression), the longitudinal strain modes \( \varphi_{i(s)} \) consist of bending about y- and z-axes, and warping in the longitudinal direction. The functions \( \varphi_{i(s)} \) are chosen in the forms:
\[ \varphi_{1(s)} = y(s); \varphi_{2(s)} = z(s); \varphi_{3(s)} = \omega_{M(s)} \]

The transverse strain modes \( \psi_{i(s)} \) consist of bending about y- and z-axes, pure rotation about the longitudinal x-axis and distortion of the cross section. The functions \( \psi_{i(s)} \) are defined as follows:
\[
\begin{align*}
\psi_1(s) &= \phi_1'(s) = y'(s); \\
\psi_2(s) &= \phi_2'(s) = z'(s); \\
\psi_3(s) &= \psi_3(s) = h(s); \\
\psi_4(s) &= \phi_4'(s) = \omega'(s).
\end{align*}
\]

The elements of the coefficient matrix of the governing differential equation of equilibrium were determined for the cross sections by first generating and plotting the strain fields as shown in figure 2. Diagram multiplication technique was then used in determining the elements from the strain mode diagrams as follows:
Fig. 2. Generalized strain fields for single-cell doubly symmetric section

\[ a_{ij} = a_{ji} = \int_s \varphi_i(s)\varphi_j(s) t(s) \, ds \]

\[ a_{11} = \int_s \varphi_1(s)\varphi_1(s) t(s) \, ds = 7.333a^3 t \]

\[ a_{22} = \int_s \varphi_2(s)\varphi_2(s) t(s) \, ds = 13.5a^3 t \]

\[ a_{12} = a_{21} = \int_s \varphi_1(s)\varphi_2(s) t(s) \, ds = 0 \]

\[ a_{13} = a_{31} = \int_s \varphi_1(s)\varphi_3(s) t(s) \, ds = 0 \]

\[ a_{23} = a_{32} = \int_s \varphi_2(s)\varphi_3(s) t(s) \, ds = 0 \]

\[ a_{33} = \int_s \varphi_3(s)\varphi_3(s) t(s) \, ds = 0.3a^3 t \]

\[ b_{ij} = b_{ji} = \int_s \varphi'_i(s)\varphi'_j(s) t(s) \, ds \]

\[ b_{11} = \int_s \varphi'_1(s)\varphi'_1(s) t(s) \, ds = 4at \]

\[ b_{12} = b_{21} = \int_s \varphi'_1(s)\varphi'_2(s) t(s) \, ds = 0 \]

\[ b_{13} = b_{31} = \int_s \varphi'_1(s)\varphi'_3(s) t(s) \, ds = 0 \]

\[ b_{22} = \int_s \varphi'_2(s)\varphi'_2(s) t(s) \, ds = 6at \]

\[ b_{23} = b_{32} = \int_s \varphi'_2(s)\varphi'_3(s) t(s) \, ds = 0 \]

\[ b_{33} = \int_s \varphi'_3(s)\varphi'_3(s) t(s) \, ds = 0.6a^3 t \]

\[ c_{ir} = c_{ri} = \int_s \varphi'_i(s)\psi_r(s) t(s) \, ds \]

\[ c_{11} = \int_s \varphi'_1(s)\psi_1(s) t(s) \, ds = 4at \]

\[ c_{12} = c_{21} = \int_s \varphi'_1(s)\psi_2(s) t(s) \, ds = 0 \]

\[ c_{13} = c_{31} = \int_s \varphi'_1(s)\psi_3(s) t(s) \, ds = 0 \]

\[ c_{14} = \int_s \varphi'_1(s)\psi_4(s) t(s) \, ds = 0 \]

\[ c_{22} = \int_s \varphi'_2(s)\psi_2(s) t(s) \, ds = 6at \]

\[ c_{23} = \int_s \varphi'_2(s)\psi_3(s) t(s) \, ds = 0 \]

\[ c_{24} = \int_s \varphi'_2(s)\psi_4(s) t(s) \, ds = 0 \]

\[ c_{33} = \int_s \varphi'_3(s)\psi_3(s) t(s) \, ds = 0.6a^3 t \]

\[ c_{34} = \int_s \varphi'_3(s)\psi_4(s) t(s) \, ds = 0.6a^3 t \]

\[ m_{rr} = m_{kr} = \int_s \psi_r(s)\psi_r(s) t(s) \, ds \]

\[ m_{11} = \int_s \psi_1(s)\psi_1(s) t(s) \, ds = 4at \]

\[ m_{12} = m_{21} = \int_s \psi_1(s)\psi_2(s) t(s) \, ds = 0 \]

\[ m_{13} = m_{31} = \int_s \psi_1(s)\psi_3(s) t(s) \, ds = 0 \]

\[ m_{14} = m_{41} = \int_s \psi_1(s)\psi_4(s) t(s) \, ds = 0 \]

\[ m_{22} = \int_s \psi_2(s)\psi_2(s) t(s) \, ds = 6at \]

\[ m_{23} = m_{32} = \int_s \psi_2(s)\psi_3(s) t(s) \, ds = 0 \]

\[ m_{24} = m_{42} = \int_s \psi_2(s)\psi_4(s) t(s) \, ds = 0 \]

\[ m_{33} = \int_s \psi_3(s)\psi_3(s) t(s) \, ds = 15a^3 t \]

\[ m_{34} = \int_s \psi_3(s)\psi_4(s) t(s) \, ds = 0.6a^3 t \]

\[ m_{44} = \int_s \psi_4(s)\psi_4(s) t(s) \, ds = 0.6a^3 t \]

\[ h_{kr} = h_{rk} = \int_s \psi_k(s)\psi_r(s) t(s) \, ds \]
\[ h_{11} = \int_s \psi_1(s) \psi_1(s) ds = m_1/t = 4a \]
\[ h_{12} = h_{21} = \int_s \psi_1(s) \psi_2(s) ds = 0 \]
\[ h_{13} = h_{31} = \int_s \psi_1(s) \psi_3(s) ds = 0 \]
\[ h_{14} = h_{41} = \int_s \psi_1(s) \psi_4(s) ds = 0 \]
\[ h_{22} = \int_s \psi_2(s) \psi_2(s) ds = 6a \]
\[ h_{23} = h_{32} = \int_s \psi_2(s) \psi_3(s) ds = 0 \]
\[ h_{24} = h_{42} = \int_s \psi_2(s) \psi_4(s) ds = 0 \]
\[ h_{33} = \int_s \psi_3(s) \psi_3(s) ds = 15a^3 \]
\[ h_{34} = \int_s \psi_3(s) \psi_4(s) ds = 0.6a^3 \]
\[ h_{44} = \int_s \psi_4(s) \psi_4(s) ds = 0.6a^3 \]
\[ s_{kr} = s_{rk} = \frac{1}{E} \int_s \frac{M_k(s) M_r(s)}{EI} ds \]
\[ s_{44} = \frac{1}{E} \int_s \frac{M_4(s) M_4(s)}{EI} ds = \frac{0.768It}{a} \]

But, \( I = \frac{t^2}{12} \) for all the plates

\[ \Rightarrow s_{44} = \frac{0.768t}{a} \times \frac{t^3}{12} = \frac{0.064t^4}{a} \]

**DERIVATION OF INDEPENDENT EQUATIONS IN LATERAL DISPLACEMENT QUANTITIES \( V_k(x) \):**

Substituting the zero coefficients as obtained into the matrix form of the governing equations of equilibrium (14 & 15), and assuming the cross-section to be rigid (non-deformable), we obtained:

\[
\begin{bmatrix}
\frac{a_{11}}{} & 0 & 0 \\
0 & \frac{a_{22}}{} & 0 \\
0 & 0 & \frac{a_{33}}{}
\end{bmatrix}
\begin{bmatrix}
U_1' \\
U_2' \\
U_3'
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{b_{11}}{} & 0 & 0 \\
0 & \frac{b_{22}}{} & 0 \\
0 & 0 & \frac{b_{33}}{}
\end{bmatrix}
\begin{bmatrix}
U_1 \\
U_2 \\
U_3
\end{bmatrix}
\]

\[
\begin{bmatrix}
c_{11} & 0 & 0 \\
0 & c_{22} & 0 \\
0 & 0 & c_{33}
\end{bmatrix}
\begin{bmatrix}
V_1' \\
V_2' \\
V_3'
\end{bmatrix}
= 0
\]

\[
\begin{bmatrix}
c_{11} & 0 & 0 \\
0 & c_{22} & 0 \\
0 & 0 & c_{33}
\end{bmatrix}
\begin{bmatrix}
U_1' \\
U_2' \\
U_3'
\end{bmatrix}
+ \begin{bmatrix}
k_{11} & 0 & 0 \\
0 & k_{22} & 0 \\
0 & 0 & k_{33}
\end{bmatrix}
\begin{bmatrix}
V_1' \\
V_2' \\
V_3'
\end{bmatrix}
= 0
\]

(19)

Where,

\[ k_{11} = \left( m_{11} - \frac{P}{G} h_{11} \right); \]

\[ k_{22} = \left( m_{22} - \frac{P}{G} h_{22} \right); \]

etc.

Expanding equation (18), we obtained:
Expanding equation (19), we obtained:

\[
\begin{align*}
\gamma a_{11} U_1'' - b_{11} U_1' - c_{11} V_1' &= 0 \\
\gamma a_{22} U_2'' - b_{22} U_2' - c_{22} V_2' &= 0 \\
\gamma a_{33} U_3'' - b_{33} U_3' - c_{33} V_3' &= 0
\end{align*}
\]

Expanding equation (19), we obtained:

\[
\begin{align*}
\gamma a_{11} U_1'' + k_{11} V_1'' &= 0 \\
\gamma a_{22} U_2'' + k_{22} V_2'' &= 0 \\
\gamma a_{33} U_3'' + k_{33} V_3'' &= 0
\end{align*}
\]

Eliminating \( U'(x) \) and their derivatives from equations (20) and (21), we obtained:

\[
\begin{align*}
v_1'' + a_{11} v_1'' &= 0 \\
v_2'' + a_{22} v_2'' &= 0 \\
v_3'' + a_{33} v_3'' &= 0
\end{align*}
\]

Where,

\[
\begin{align*}
a_{11} &= \frac{c_{11}^2 - b_{11} k_{11}}{\gamma a_{11} k_{11}} \\
a_{22} &= \frac{c_{22}^2 - b_{22} k_{22}}{\gamma a_{22} k_{22}} \\
a_{33} &= \frac{c_{33}^2 - b_{33} k_{33}}{\gamma a_{33} k_{33}}
\end{align*}
\]

The independence of the above three equations shows that there are three possible independent buckling modes, namely: flexural buckling about \( oz \)- and \( oy \)-axes (22) and (23) and torsional buckling mode about \( ox \)-axis (24). The general solution of the flexural modes (22) and (23) are given by:

\[
\begin{align*}
V_1 &= C_1 \cos \alpha_1 x + C_2 \sin \alpha_1 x + C_3 x + C_4 \\
V_2 &= C_1 \cos \alpha_2 x + C_2 \sin \alpha_2 x + C_3 x + C_4
\end{align*}
\]

The constants \( C_1 \), \( C_2 \), \( C_3 \) and \( C_4 \) were evaluated from the boundary conditions as follows:

(i) Hinged-Hinged condition:

\[
\begin{align*}
\frac{d^2 V_1}{dx^2} &= \frac{d^2 V_2}{dx^2} = 0 (x = 0, I)
\end{align*}
\]

(ii) Clamped–Hinged condition:

\[
\begin{align*}
\frac{d V_1}{dx} &= \frac{d V_2}{dx} = 0 (x = 0) \\
\frac{d^2 V_1}{dx^2} &= \frac{d^2 V_2}{dx^2} = 0 (x = I)
\end{align*}
\]

(iii) Clamped–Clamped condition:

\[
\begin{align*}
V_1 &= V_2 = 0; \\
\frac{d V_1}{dx} &= \frac{d V_2}{dx} = 0 (x = 0, I)
\end{align*}
\]

Applying the boundary conditions (27), (28) and (29) to equations (25) and (26) and noting that for nontrivial solutions or nonzero values of the constants, the determinant of the coefficients of \( C_1 \) ... \( C_4 \) must vanish, we obtained the following:

(i) Hinged-Hinged conditions:-
and
\[
\begin{bmatrix}
1 & 0 & 0 & 1 \\
0 & a_{11} & 1 & 0 \\
\cos \alpha_{11} & \sin \alpha_{11} & l & 1 \\
\cos \alpha_{11} & \sin \alpha_{11} & 0 & 0
\end{bmatrix}
= 0
\]

(ii) Clamped–Hinged conditions:
\[
\begin{bmatrix}
1 & 0 & 0 & 1 \\
0 & a_{22} & 1 & 0 \\
\cos \alpha_{22} & \sin \alpha_{22} & l & 1 \\
\cos \alpha_{22} & \sin \alpha_{22} & 0 & 0
\end{bmatrix}
= 0
\]

(iii) Clamped-Clamped conditions:
\[
\begin{bmatrix}
1 & 0 & 0 & 1 \\
0 & a_{11} & 1 & 0 \\
-\alpha_{11} \sin \alpha_{11} & a_{11} \cos \alpha_{11} & l & 1 \\
\end{bmatrix}
= 0
\]

Equations (30), (31) and (32) represent the stability matrices for equations (22) and (23) for the two axes of symmetry and the different boundary conditions. Expanding equations (30), (31), and (32), we obtained the critical buckling loads for the respective boundary conditions as follows:

(i)
\[
P_{cr} = \frac{m_{11} - \frac{c_{11}^2}{n^2 \pi^2}}{\frac{u_{11}^2}{l^2} + \gamma a_{11} + b_{11}} G \frac{h_{11}}{h_{11}}
\]

(ii)
\[
P_{cr} = \frac{m_{11} - \frac{c_{11}^2}{20.19}}{\frac{u_{11}^2}{l^2} + \gamma a_{11} + b_{11}} G \frac{h_{11}}{h_{11}}
\]

(iii)
\[
P_{cr} = \frac{m_{11} - \frac{c_{11}^2}{4n^2 \pi^2}}{\frac{u_{11}^2}{l^2} + \gamma a_{11} + b_{11}} G \frac{h_{11}}{h_{11}}
\]
A numerical study was performed for single-cell thin-walled steel box column with the following parameters:

- $E = 210 \times 10^3\text{MN/m}^2$
- $G = 81 \times 10^3\text{MN/m}^2$
- $L = 4.5\text{m}$
- $a = 0.08\text{m}$
- $t = 0.0005\text{m}$ to $0.02\text{m}$

The critical loads associated with the two flexural buckling modes were evaluated for the different boundary conditions and the results presented on tables 1 and 2.

### THE EULER BUCKLING LOAD EQUIVALENT:

The Euler’s buckling load is given by:

$$P_{cr} = \frac{\pi^2 EI}{l_{eff}^2}$$

**Bending about oz-axis:** $I_z = a_{11} = 7.333a^3t$, $a = 0.08m$, $L = 4.5m$

- Hinged-hinged condition: $l_{eff} = L = 4.5m$; \[P_{cr} = \frac{\pi^2 EI}{l_{eff}^2} = 384.278t\]
- Clamped-clamped condition: $l_{eff} = 0.7L$; \[P_{cr} = 784.241t\]
- Clamped-clamped condition: $l_{eff} = 0.5L$; \[P_{cr} = 1537.112t\]

**Bending about oy-axis:** $I_y = 13.5a^3t$

- Hinged-hinged condition:
- Clamped-clamped condition:
- Clamped-clamped condition:

Again, the critical loads associated with the two flexural buckling modes were evaluated for the different boundary conditions and the results presented on tables 1 and 2.

### RESULTS AND DISCUSSION

The results as presented on tables (1 and 2) show that the flexural buckling load about oz-axis is less than that about oy-axis. Hence, the flexural behaviour shows that for all the three sets of boundary conditions considered, the critical load due to flexure about the oz-axis will control the design. Comparison of Vlasov critical load values in table 1 with the Euler critical load results showed that the Euler model underestimated the critical buckling loads by 67.53% for hinged-hinged, 67.11% for clamped-hinged and 66.11% for clamped-clamped boundary conditions under flexure about the oz-axis. However, under flexure about oy-axis, the percentages of underestimation by Euler model were reduced to 51.14% for hinged-hinged, 50.33% for clamped-hinged and 48.52% for clamped-clamped boundary conditions respectively. In both axes, the results also indicate that improved fixity reduces the degree of underestimation by Euler model.
The results generally show that for single-cell doubly symmetric box columns, the Euler buckling strength could be used to estimate the Vlasov flexural buckling strength by increasing the Euler value by about 100% to 200% depending on the axis of symmetry and the boundary conditions.

**CONCLUSION**

This study has resulted in a better understanding of the level of difference between the more accurate Vlasov critical loads for single-cell thin-walled box columns and the underestimated Euler critical loads. The results confirms that Euler's critical load model is indeed inadequate for instability design of thin-walled box columns. A critical comparison of the results show that the simpler Euler flexural buckling loads can now be used for making a reasonable estimate of the more accurate and more rigorous Vlasov flexural buckling value.

**Table 1:** Vlasov and Euler critical buckling loads for single-cell doubly symmetric thin-walled box column under the different boundary conditions and bending about oz-axis

<table>
<thead>
<tr>
<th>t(m)</th>
<th>Pcr(Vlasov)</th>
<th>Pcr (Euler)</th>
<th>Pcr(Vlasov)</th>
<th>Pcr (Euler)</th>
<th>Pcr(Vlasov)</th>
<th>Pcr (Euler)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>23.673</td>
<td>7.686</td>
<td>47.686</td>
<td>15.685</td>
<td>90.714</td>
<td>30.742</td>
</tr>
<tr>
<td>0.0175</td>
<td>20.713</td>
<td>6.725</td>
<td>41.725</td>
<td>13.724</td>
<td>79.374</td>
<td>26.899</td>
</tr>
<tr>
<td>0.015</td>
<td>17.754</td>
<td>5.764</td>
<td>35.764</td>
<td>11.764</td>
<td>68.035</td>
<td>23.057</td>
</tr>
<tr>
<td>0.01</td>
<td>11.836</td>
<td>3.843</td>
<td>23.843</td>
<td>7.842</td>
<td>45.357</td>
<td>15.371</td>
</tr>
<tr>
<td>0.0075</td>
<td>8.877</td>
<td>2.882</td>
<td>17.882</td>
<td>5.882</td>
<td>34.018</td>
<td>11.283</td>
</tr>
<tr>
<td>0.005</td>
<td>5.918</td>
<td>1.921</td>
<td>11.921</td>
<td>3.921</td>
<td>22.678</td>
<td>7.686</td>
</tr>
<tr>
<td>0.0025</td>
<td>2.959</td>
<td>0.961</td>
<td>5.961</td>
<td>1.961</td>
<td>11.339</td>
<td>3.843</td>
</tr>
<tr>
<td>0.001</td>
<td>1.184</td>
<td>0.384</td>
<td>2.384</td>
<td>0.784</td>
<td>4.536</td>
<td>1.537</td>
</tr>
<tr>
<td>0.00075</td>
<td>0.888</td>
<td>0.288</td>
<td>1.788</td>
<td>0.588</td>
<td>3.402</td>
<td>1.153</td>
</tr>
<tr>
<td>0.0005</td>
<td>0.592</td>
<td>0.192</td>
<td>1.192</td>
<td>0.392</td>
<td>2.263</td>
<td>0.769</td>
</tr>
</tbody>
</table>
Table 2: Vlasov and Euler critical buckling loads for single-cell doubly symmetric thin-walled box column under the different boundary conditions and bending about oY-axis

<table>
<thead>
<tr>
<th>t(m)</th>
<th>Pcr(Vlasov)</th>
<th>Pcr (Euler)</th>
<th>Pcr(Vlasov)</th>
<th>Pcr (Euler)</th>
<th>Pcr(Vlasov)</th>
<th>Pcr (Euler)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>28.958</td>
<td>14.149</td>
<td>58.137</td>
<td>28.876</td>
<td>109.963</td>
<td>56.596</td>
</tr>
<tr>
<td>0.0175</td>
<td>25.338</td>
<td>12.380</td>
<td>50.87</td>
<td>25.266</td>
<td>96.194</td>
<td>49.522</td>
</tr>
<tr>
<td>0.015</td>
<td>21.718</td>
<td>10.612</td>
<td>43.603</td>
<td>21.657</td>
<td>82.452</td>
<td>42.447</td>
</tr>
<tr>
<td>0.0125</td>
<td>18.099</td>
<td>8.843</td>
<td>36.335</td>
<td>18.047</td>
<td>68.710</td>
<td>35.373</td>
</tr>
<tr>
<td>0.01</td>
<td>14.479</td>
<td>7.075</td>
<td>29.068</td>
<td>14.438</td>
<td>54.968</td>
<td>28.298</td>
</tr>
<tr>
<td>0.0075</td>
<td>10.859</td>
<td>5.306</td>
<td>21.801</td>
<td>10.828</td>
<td>41.226</td>
<td>21.224</td>
</tr>
<tr>
<td>0.005</td>
<td>7.239</td>
<td>3.537</td>
<td>14.534</td>
<td>7.219</td>
<td>27.484</td>
<td>14.149</td>
</tr>
<tr>
<td>0.0025</td>
<td>3.620</td>
<td>1.769</td>
<td>7.267</td>
<td>3.609</td>
<td>13.742</td>
<td>7.075</td>
</tr>
<tr>
<td>0.001</td>
<td>1.448</td>
<td>0.707</td>
<td>2.907</td>
<td>1.444</td>
<td>5.497</td>
<td>2.830</td>
</tr>
<tr>
<td>0.00075</td>
<td>1.086</td>
<td>0.531</td>
<td>2.180</td>
<td>1.083</td>
<td>4.123</td>
<td>2.122</td>
</tr>
<tr>
<td>0.0005</td>
<td>0.724</td>
<td>0.354</td>
<td>1.453</td>
<td>0.722</td>
<td>2.748</td>
<td>1.415</td>
</tr>
</tbody>
</table>

REFERENCES